Proton polarisability contribution to the Lamb shift in muonic hydrogen

Mike Birse

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Work done in collaboration with Judith McGovern

Eur. Phys. J. A **48** (2012) 120

- Two-photon contribution to the Lamb shift
- Low-energy theorems for doubly-virtual Compton scattering
- Calculation of subtraction term in Chiral Perturbation Theory

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The proton radius puzzle 1

Lamb shift in muonic hydrogen: $\Delta E_L = E(2p_{\frac{1}{2}}) - E(2s_{\frac{1}{2}}) \simeq +0.2$ eV

Much larger than in electronic hydrogen, dominated by vacuum polarisation and much more sensitive to proton structure , in particular, its charge radius

$$\Delta E_L^{\text{th}} = 206.0668(25) - 5.2275(10)\langle r_E^2 \rangle \text{ meV}$$

Results of many years of effort by Borie, Pachucki, Indelicato, Jentschura and others; collated in Antognini et al, Ann. Phys. **331** (2013) 127

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Includes contribution from two-photon exchange

$$\Delta E^{2\gamma} = 33.2(20)\ \mu\text{eV}$$

Sensitive to polarisabilities of proton by virtual photons

Focus of this talk

The proton radius puzzle 2

CREMA experiment at PSI: $2p_{\frac{3}{2}} \rightarrow 2s_{\frac{1}{2}}$ transitions to both hyperfine $2s$ states

Pohl et al, Nature **466** (2010) 213; Antognini et al, Science **339** (2013) 417

Eliminate hyperfine splitting to get

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CODATA 2010 value for charge radius, $r_E = 0.8775(51)$ fm (electronic H), gives

$$\Delta E_L^{\text{th}} = 202.042(47)\ \text{meV}$$

Discrepancy: $0.330(47)$ meV (7σ!)

New value for charge radius: $r_E = 0.84087 \pm 0.00026(\text{exp}) \pm 0.00029(\text{th})$ fm

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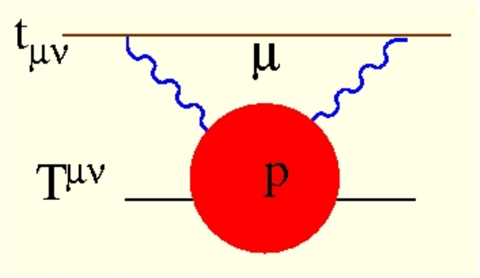
In 2010: $\Delta E^{2\gamma} \sim 0.03$ meV was least-well determined contribution to ΔE_L^{th}

Still contributes largest single uncertainty

But would need to be 10 times larger to explain the discrepancy

Two-photon exchange



Integral over $T^{\mu\nu}(\nu, q^2)$ – doubly-virtual Compton amplitude for proton

Spin-averaged, forward scattering $\longrightarrow$ two independent tensor structures

Common choice:

$$T^{\mu\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) T_1(\nu, Q^2) + \frac{1}{M^2}\left(p^\mu - \frac{p\cdot q}{q^2} q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2} q^\nu\right) T_2(\nu, Q^2)$$

multiplied by scalar functions of $\nu = p \cdot q/M$ and $Q^2 = -q^2$

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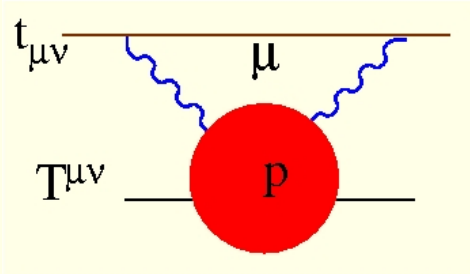

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Amplitude contains elastic (Born) and inelastic pieces: $T^{\mu\nu} = T_B^{\mu\nu} + \overline{T}^{\mu\nu}$

- elastic: photons couple independently to proton (no excitation)
- need to remove terms already accounted for in Lamb shift (iterated Coulomb, leading dependence on $\langle r_E^2\rangle$)
- inelastic: proton excited $\longrightarrow$ polarisation effects

Doubly-virtual Compton scattering

Elastic amplitude from Dirac nucleon with Dirac and Pauli form factors

K. Pachucki, Phys. Rev. A **60** (1999) 3593

$$\Gamma^\mu = F_D(q^2)\gamma^\mu + iF_P(q^2)\frac{\sigma^{\mu\nu}q^\nu}{2M}$$

Gives

$$T_1^B(\nu, Q^2) = \frac{e^2}{M}\left[\frac{Q^4\left(F_D(Q^2)+F_P(Q^2)\right)^2}{Q^4-4M^2\nu^2} - F_D(Q^2)^2\right]$$

$$T_2^B(\nu, Q^2) = \frac{4e^2MQ^2}{Q^4-4M^2\nu^2}\left[F_D(Q^2)^2 + \frac{Q^2}{4M^2}F_P(Q^2)^2\right]$$

Final term in T_1 – no pole corresponding to on-shell intermediate nucleon

But this depends on choice of tensor basis (energy-dependent tensors)

cf Walker-Loud et al, Phys Rev Lett **108** (2012) 232301

Also parts of this term required by low-energy theorems (eg Thomson limit)

$\longrightarrow$ keep it as part of Born amplitude

Low-energy theorems

V^2CS not directly measurable, but constrained by LETs

Expand in tensor basis without kinematic singularities $(1/q^2)$

Tarrach, Nuov Cim **28A** (1975) 409

$\longrightarrow$ two independent tensors of order q^2: correspond to polarisabilities $\alpha+\beta$ and β from real Compton scattering

$$\begin{aligned}\overline{T}_1(\omega,Q^2) &= 4\pi Q^2\beta + 4\pi\omega^2(\alpha+\beta) + O(q^4)\\ \overline{T}_2(\omega,Q^2) &= 4\pi Q^2(\alpha+\beta) + O(q^4)\end{aligned}$$

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Nonpole term in Born amplitude T_1^B contains piece $\propto Q^2$:

$$F_D(Q^2)^2 = 1 - \left[\frac{1}{3}\langle r_E^2\rangle - \frac{\kappa}{2M^2}\right]Q^2 + O(Q^4)$$

Moving this to inelastic amplitude would require modified LET

(if β defined in usual way from real Compton scattering)

All these LETs automatically built into EFTs at 4th order (NRQED, HBChPT)

eg Hill and Paz, Phys Rev Lett **107** (2011) 160402

Dispersion relations

Information on forward V^{2}CS away from $q = 0$ from structure functions $F_{1,2}(\nu, Q^2)$ via dispersion relations

$$\overline{T}_2(\nu, Q^2) = -\int_{\nu_{th}^2}^{\infty} \mathrm{d}\nu'^2 \, \frac{F_2(\nu', Q^2)}{\nu'^2 - \nu^2}$$

– integral converges since $F_2 \sim 1/\nu$ at high energies

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But $F_1 \sim \nu$ so need to use subtracted dispersion relation

$$\overline{T}_1(\nu, Q^2) = \overline{T}_1(0, Q^2) - \nu^2 \int_{\nu_{th}^2}^{\infty} \frac{d\nu'^2}{\nu'^2} \, \frac{F_1(\nu', Q^2)}{\nu'^2 - \nu^2}$$

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Subtraction function $\overline{T}_1(0, q^2)$ not experimentally accessible
Satisfies LET: $\overline{T}_1(0, Q^2)/Q^2 \to 4\pi\beta$ as $Q^2 \to 0$
But how does it depend on Q^2?

Subtraction term

Define form factor

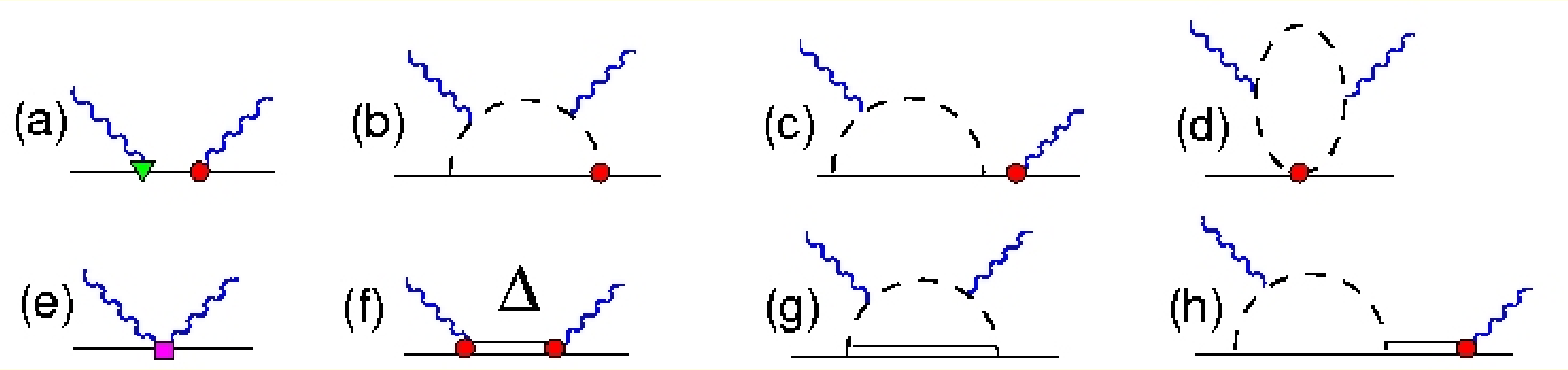
$$\overline{T}_1(0,Q^2) = 4\pi\beta\, Q^2\, F_\beta(Q^2)$$

Large Q^2: operator-product expansion, quark counting rules give $F_\beta(Q^2) \propto Q^{-4}$

Small Q^2: use HBChPT at 4th order, plus leading effect of $\gamma N\Delta$ form factor

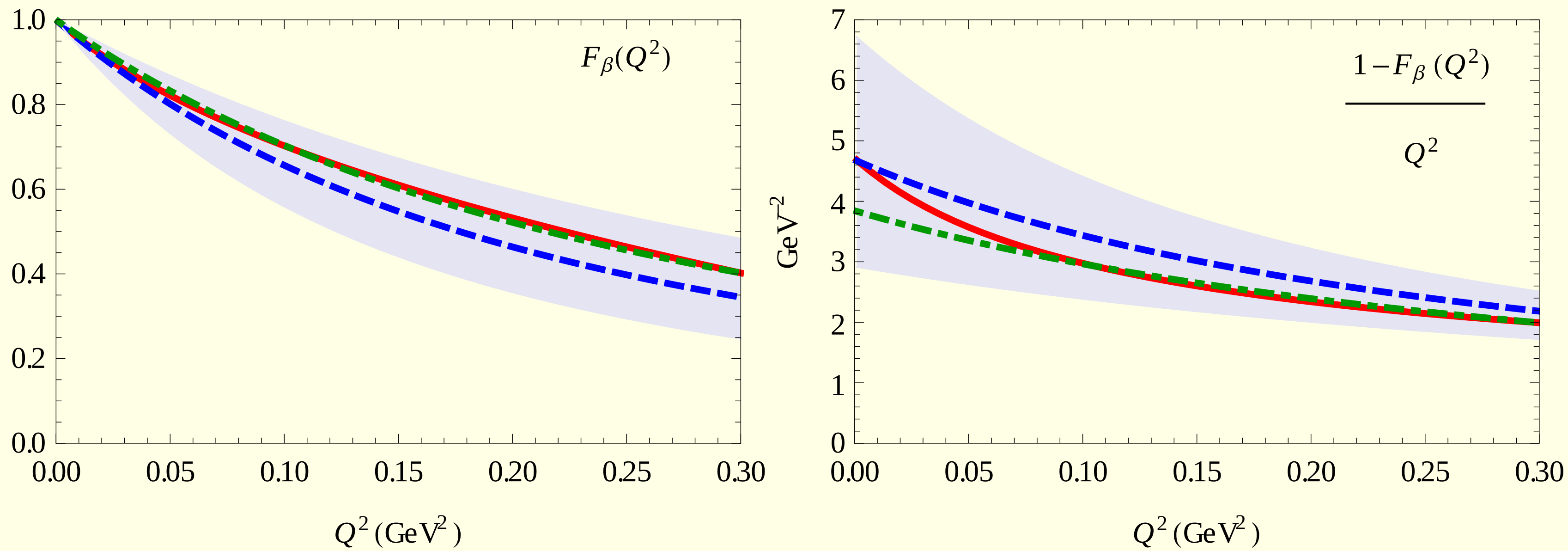
- same diagrams as for real Compton scattering

McGovern et al, Eur. Phys. J. A 49 (2013) 12



- minor modifcations for different kinematics
- subtract elastic (Born) contribution calculated to this order

Form factor



Extrapolate to higher Q^2 by matching ChPT form onto dipole

$$F_\beta(Q^2) \sim \frac{1}{(1+Q^2/2M_\beta^2)^2}$$

Match at $Q^2 = 0 \to M_\beta = 462$ MeV; at $Q^2 \sim m_\pi^2 \to M_\beta = 510$ MeV

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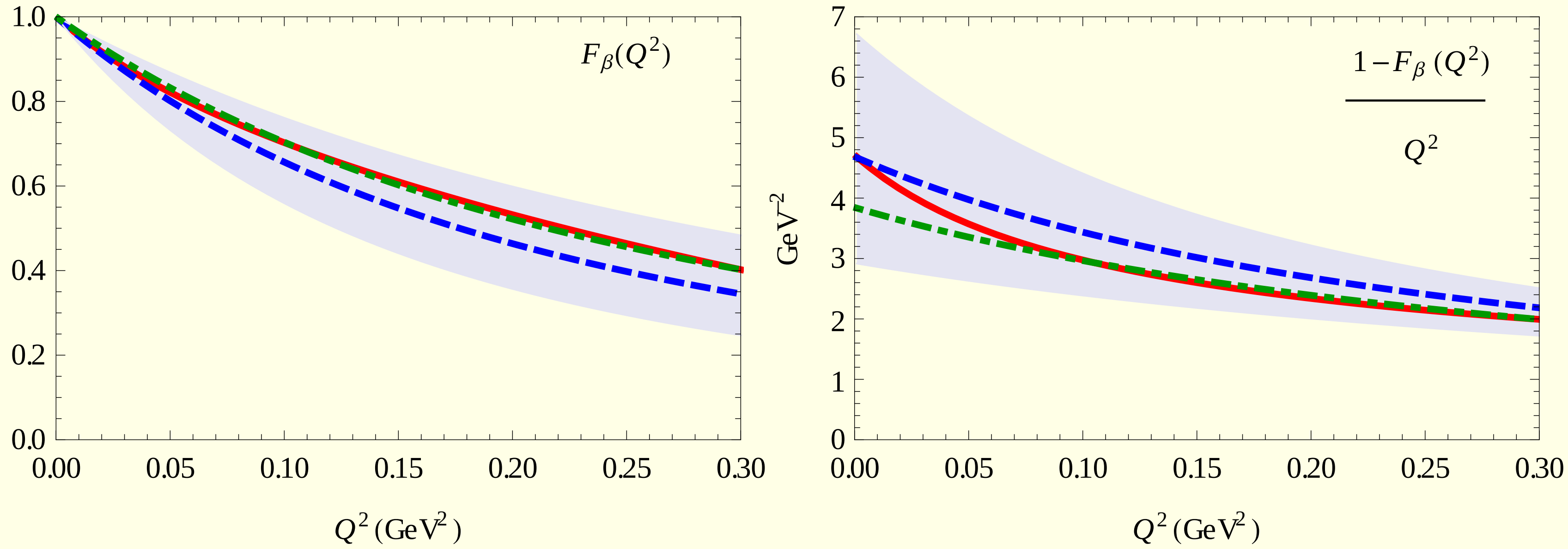

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$$M_\beta = 485 \pm 100 \pm 40 \pm 25 \text{ MeV}$$

- generous allowance for higher-order effects and uncertainties in input (shaded)
- $\beta = (3.1 \pm 0.5) \times 10^{-4}$ fm^3
- matching uncertainty

Energy shift

$$\Delta E^{2\gamma}_{\rm sub} = \frac{\alpha_{\rm EM}\,\phi(0)^2}{4\pi m}\int_0^\infty {\rm d}Q^2\,\frac{\overline{T}_1(0,Q^2)}{Q^2}\times\left[1+\left(1-\frac{Q^2}{2m^2}\right)\left(\sqrt{\frac{4m^2}{Q^2}+1}-1\right)\right]$$

- with dipole form, 90% comes from $Q^2 < 0.3$ GeV^2
- rather insensitive to value of M_β
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Result:

$$\Delta E^{2\gamma}_{\rm sub} = -4.2 \pm 1.0\,\mu\text{eV}$$

Comparable to previous, model-based results Pachucki, Phys. Rev. A **60** (1999) 3593; Carlson and Vanderhaeghen, Phys. Rev. A **84** (2011) 020102

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Combined with results of Carlson and Vanderhaeghen

- elastic (with nonpole term reinstated): $\Delta E^{2\gamma}_{\rm el} = 24.7(13)\ \mu$eV
- inelastic (dispersive): $\Delta E^{2\gamma}_{\rm inel} = 12.7(5)\ \mu$eV

$\rightarrow$ total: $\Delta E^{2\gamma} = 33.2(20)\ \mu$eV

Extrapolation questions 1

Extrapolation not needed in ChPT at 3rd order – two-photon loop finite
$\longrightarrow$ calculate $\Delta E^{2\gamma}$ directly Nevado and Pineda, Phys Rev C **77** (2008) 035202
(ChPT with added leptons – needs lepton-nucleon contact terms at higher orders)

ChPT at 4th order
- consistent with current determination of magnetic polarisability β
- lowest order that makes direct contact with LETs
- but form factors unphysical above breakdown scale $\longrightarrow$ extrapolate

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nucleons become very soft for momentum scales $Q^2 \gtrsim 2$ GeV^2
Miller, Phys Lett B **718** (2013) 1078

Extrapolation questions 2

But no evidence from related processes:

- dispersion relations for $T_2(0, Q^2)$ ($\sim \alpha + \beta$)
- proton-neutron mass difference Walker-Loud et al, Phys Rev Lett **108** (2012) 232301
- quasi-elastic electron-nucleus scattering Miller, Phys Rev C **86** (2012) 065201

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Nor from energy-weighted sum rules (despite large uncertainties)

Gorchtein et al, Phys Rev A **87** (2013) 052501

- after transfer of nonpole Born term back to elastic piece

$$\Delta E^{2\gamma}_{\rm sub} = +1.5 \pm 4.6\,\mu\text{eV}$$

(opposite sign for central value since $\beta = -0.3 \pm 4.0$)

Summary

Subtraction term in two-photon-exchange contribution to Lamb shift calculated using HBChPT at 4th order

$$\Delta E^{2\gamma}_{\rm sub} = -4.2 \pm 1.0\,\mu\text{eV}$$

Complete two-photon exchange contribution now well determined

$$\Delta E^{2\gamma} = 33 \pm 2\,\mu\text{eV}$$

- factor 10 too small to explain proton radius puzzle (330 μeV)
- extrapolation of ChPT result needed at 4th order
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- main sources of uncertainty: β (subtraction) and form factors (elastic)

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Important to maintain consistency between definition of elastic/Born contribution and LET satisfied by subtraction term

- form of nucleon pole terms depends on choice of tensor basis
- leading two terms in nonpole piece of $T_1(0, Q^2)$ both constrained by LETs

$\rightarrow$ take full Born terms for Dirac nucleon in elastic contribution to Lamb shift