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Issues with determining the proton radius from elastic electron scattering

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Background

Long history of discrepant results for proton charge radius from elastic electron scattering [PDG, Phys Rev **D86** (2012) 01001]

- pre-1980 values: $r_E \sim 0.80 - 0.88$ fm
- recent (post-1990) $r_E \sim 0.84 - 0.91$ fm
  some from analyses of same data!
- support for either of Lamb shift values

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Problem: radius given by slope of electric form factor at $Q^2 = 0$

$\longrightarrow$ need to extrapolate from data at finite Q^2

- older data: relatively large values of Q^2
- long extrapolation based on fits to region "where the light is good"

Possible solutions

Theory: use fitting functions that contain the correct physics controlling behaviour at small Q^2

- dispersion relations, chiral perturbation theory
- examples: Mergell *et al*, Nucl Phys **A596** (1996) 367; Lorenz *et al*, arXiv:1205.6628; Hill and Paz, Phys Rev **D82** (2010) 113005

$\longrightarrow$ results of fits by different groups barely consistent

$r_E \sim 0.84 \pm 0.01 - 0.87 \pm 0.01$ fm

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Experiment: take data at much smaller Q^2

- now available from A1@MAMI, down to $Q^2 \simeq 0.004$ GeV^2
  Bernauer *et al*, Phys Rev Lett **105** (2010) 242001
- but . . .

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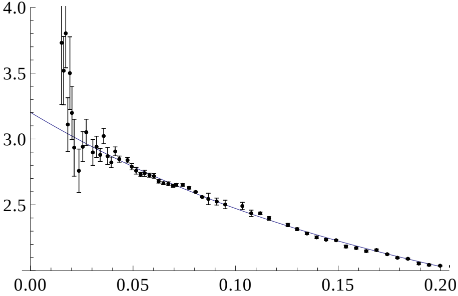
MAMI data

Rosenbluth separation of MAMI data, $Q^2 > 0.015\ \mathrm{GeV}^2$

[Bernauer, PhD thesis, Mainz, 2010]

Health warning: large spectrometer acceptances $\to$ systematic effects not fully accounted for in error bars

Plot $\dfrac{1 - G_E(Q^2)}{Q^2}$

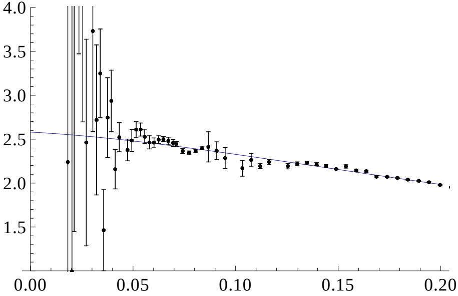


Fit: 5th-order polynomial in Q^2 to data $0.02 \leq Q^2 \leq 0.55\ \mathrm{GeV}^2$

- $G'_E(0) = -3.202\ \mathrm{GeV}^{-2} \to r_c = 0.865\ \mathrm{fm}$
- $\chi^2/\mathrm{dof} = 2.15$ (72 data points, 5 parameters)

Magnetic form factor

Plot $\frac{1 - G_M(Q^2)}{Q^2}$



Fit: 6th-order polynomial in Q^2 to data $0.02 \leq Q^2 \leq 0.55\ \text{GeV}^2$

- $G'_M(0) = -2.581\ \text{GeV}^{-2} \longrightarrow r_M = 0.776\ \text{fm}$
- $\chi^2/\text{dof} = 1.97$ (72 data points, 6 parameters)

(A1 average of fits: $r_E = 0.879 \pm 0.008$ fm, $r_M = 0.777 \pm 0.017$ fm)

What could go wrong?

Significant curvature of "$G'_{E,M}(Q^2)$" in region below $Q^2 \simeq 0.02$ GeV^2

Possible sources

- experimental: normalisation of data
- analysis: two-photon exchange correction
- physics: two-pion cut at $Q^2 = -0.078$ GeV^2 (pion cloud)
- fit: overfitting the data

Pion cloud

Photon can couple to two pions, threshold at $t = -Q^2 = 4m_\pi^2$

$\longrightarrow$ nonanalytic functions of $Q^2/(4m_\pi^2)$ in form factors

- cannot be well approximated by smooth functions of Q^2 (eg polynomials)

Chiral perturbation theory

- πN loop diagrams start at order $\mathcal{O}(p^3)$ in heavy-baryon ChPT [Bernard *et al*, Nucl Phys **A635** (1998) 121]
- $\mathcal{O}(p^4)$ corrections contained in relativistic approach [Kubis and Meissner, Nucl Phys **A679** (2001 698]
- $\pi\Delta$ loops could also be large [Bernard *et al*]

Effect of including corrections up to $\mathcal{O}(p^4)$

- nonanalytic terms from heavy-baryon expansion of expressions given by Kubis and Meissner
- $r_E \rightarrow r_E + 0.002$ fm, $r_M \rightarrow r_M + 0.004$ fm
  ($\mathcal{O}(p^3)$ slightly larger)
- $\pi\Delta$ loops change radii by less than 0.0005 fm
  (large but $\sim$ completely absorbed by refitting polynomial)
- overall effect on extrapolation small
  (cf dispersion relation with $\pi\pi$ cut, Hill and Paz, Phys Rev **D82** (2010) 113005)

Two-photon exchange

Small ($\mathcal{O}(\alpha)$) but two-photon cut starts at $t = 0$

- nonanalytic behaviour could be important for very small Q^2

$\longrightarrow$ need to remove it form measured cross sections

- MAMI data corrected only by dividing out Coulomb correction: $Q^2 = 0$ limit of correction to G_E treated as an overall factor $1 + \delta_C$ in cross section, where

$$\delta_C = \alpha\pi \frac{\sqrt{1-\varepsilon}}{\sqrt{1+\varepsilon}+\sqrt{1-\varepsilon}}$$

$\left(\varepsilon = \left[1 + 2(1 + Q^2/(4M^2))\tan^2\theta/2\right]^{-1}\right)$

Full 2γ contributions as corrections to E, M form factors at low Q^2
[Borisyuk and Kobushkin, Phys Rev **C75** (2007) 038202]

- complicated expressions but can be evaluated analytically assuming dipole forms for form factors
- expand $\delta_C(\varepsilon)$, $\delta G_{E,M}(Q^2,\varepsilon)$ to order ε since data already Rosenbluth separated
- reinstate $\delta_C/2$, subtract off B&K $\delta G_{E,M}$
- $r_E \to r_E - 0.005$ fm, $\qquad r_M \to r_M + 0.023$ fm
  (similar to effects found in reanalysis by Bernauer *et al*, Phys Rev Lett **107** (2011) 119102)

Normalisation

A1@MAMI: lot of effort into determining normalisation of data

- important: forcing a fit to give 1 as $Q^2 \to 0$ when data does not would introduce significant curvature in "$G'_E(Q^2)$" at small Q^2
- float normalisation: fit to $G_E(Q^2)(1+\delta N)$ with δN as a parameter
- $r_E \to r_E + 0.007 = 0.869$ fm, $\quad \delta N = -0.0020$
  (cf spread of normalisation constants from A1@MAMI: 0.0026)
  $\chi^2/\text{dof} = 2.19$
- refit $G_M(Q^2)(1+\delta N)$ with δN from fit to G_E
- $r_M \to r_M + 0.008 = 0.811$ fm
  $\chi^2/\text{dof} = 2.11$

Choice of fit

Polynomial functions for low Q^2 (motivated by ChPT)

$$G(Q^2) = 1 - \sum_{k=1}^{K} a_k Q^{2k} + \text{nonanalytic terms}$$

Vary order of polynomial, check

- χ^2/dof
- Akaike information criterion: $A_c = \chi^2 + 2K + \dfrac{2K(K+1)}{N-K-1}$

  N: data, K: parameters
- "naturalness" of coefficients on scale $\sim 0.5\ \text{GeV}^2$
- stability of low-order coefficients against changing K
- stability against including $Q^2 < 0.02\ \text{GeV}^2$, excluding $Q^2 > 0.4\ \text{GeV}^2$

$\longrightarrow$ $K = 5, 6$ fits of similar quality for both $G_{E,M}$

Error estimates

Problems with estimating errors since minimum $\chi^2_{\rm min}/{\rm dof} \sim 2$
(health warning on Rosenbluth separation)

- assume errors on data under-estimated, random
- use ellipsoids where $\chi^2 = \chi^2_{\rm min} + \chi^2_{\rm min}/{\rm dof}$
  (instead of $\chi^2_{\rm min} + 1$)

$\rightarrow$ $r_E = 0.869 \pm 0.009$ fm (5th-order polynomial)
$\delta N = -0.0020 \pm 0.0024$
r_E and δN very strongly correlated
$r_M = 0.811 \pm 0.008 \pm 0.009$ fm (6th-order)
first error: fit to G_M, second: δN

Sanity check: cross sections at very low Q^2

Compare total cross sections with these form factors and two-photon exchange to data for $0.004 < Q^2 < 0.02$ GeV^2 (not fitted)

- $\chi^2/N = 2.68$ (243 points)
- not good, but...

Sanity check: cross sections at very low Q^2

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- $\chi^2/N = 2.68$ (243 points)
- not good, but... very sensitive to δN
- use parameters from fits above but adjust $\delta N = +0.0002$
  or adjust δN and refit Rosenbluth data

$\longrightarrow$ $\chi^2/(N-1) = 0.95$

- fits consistent with data for low Q^2

Conclusions

Possible sources of curvature in $G'_{E,M}(Q^2)$ at low Q^2

- nonanalytic effects of pion cloud: small
- two-photon exchange: larger, important for magnetic radius
- floating normalisation: potentially very important
  (δN strongly correlated with radii)
  but in practice fairly small

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Fits to A1@MAMI Rosenbluth separated data

- $r_E = 0.869 \pm 0.009$ fm
- $r_M = 0.811 \pm 0.008 \pm 0.009$ fm
- errors underestimated because of systematics?
- also, fits to $Q^2 < 0.3$ GeV^2 unstable
- consistent with A1 refit of full dataset, after correcting for Q^2 dependence of two-photon exchange
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$\longrightarrow$ no change to radius puzzle