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Deconstructing nucleon-nucleon scattering

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Work in progress, based on:

Barford and Birse, hep-ph/0206146

Birse and McGovern, nucl-th/0307050

Birse, arXiv:0706.0984

Prompted by:

Shukla, Phillips and Mortenson, arXiv:0803.4190

Background

"Weinberg"/naive/engineering power counting for EFTs

- organise terms by counting powers of low-energy scales Q (momenta, $m_\pi, \ldots$)
- nonrelativistic NN loops of order Q (not Q^2) [Weinberg (1991)]
- theory still perturbative if potential starts at order Q^0
- cannot naturally generate low-energy bound states (or virtual states or resonances)

$\longrightarrow$ need to identify new low-energy scales

- promote leading-order terms to order Q^{-1}

$\longrightarrow$ can, and must, then be iterated to all orders

Examples of new scales

- S-wave scattering lengths $1/a \lesssim 40$ MeV [van Kolck; KSW (1998)]
- inverse Bohr radius $\kappa = \alpha M_N/2 \simeq 3.5$ MeV (*pp* scattering)
- strength of OPE set by scale

$$\lambda_{NN} = \frac{16\pi F_\pi^2}{g_A^2 M_N} \simeq 290 \text{ MeV}$$

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General tool to analyse dependence on low-energy scales and determine power counting: renormalisation group

Renormalisation group

Outline derivation of RG equation

- identify all relevant low-energy scales
- regulate at arbitary scale Λ (cut-off or subtraction) between Q and Λ_0: scale of underlying physics
- demand that physics be independent of Λ (eg T-matrix)
- rescale: express all dimensioned quantities in units of Λ (MV and all low-energy scales)

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Follow flow as Λ runs down from Λ_0

$\longrightarrow$ tends to fixed points (scale free systems) as $\Lambda \to 0$

- expand around these: perturbations scale like Λ^ν where $\nu = d + 1$ (usual power counting: Q^d)

Solution to RG equation for pure short-range potential

$$\frac{1}{V(p,\Lambda)} = -\frac{M}{2\pi^2}\left[\Lambda - \frac{p}{2}\ln\frac{\Lambda+p}{\Lambda-p}\right] - \frac{M}{4\pi}\left[-\frac{1}{a} + \frac{1}{2}r_e p^2 + \cdots\right]$$

- first term: fixed point of RG (bound state at zero energy) [Birse, McGovern, Richardson, hep-ph/9807302]
- RG eigenvalues $\nu = -1, +1, \ldots$ correspond to Q^{-2}, Q^0, ... (shifted by -2 from naive)
- coefficients of perturbations directly related to observables: effective-range expansion [Bethe (1949)]
- power counting for potential $\rightarrow$ counting for observables
- expansion in powers of energy (p^2) breaks down at $p = \Lambda$

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Similar results in presence of long-range potentials

- power counting determined by singularity of potential as $r \rightarrow 0$

Extracting a potential from empirical phase shifts (or, one day, data?)

- first determine power counting using the RG
- then use results to guide analysis:
  indicate which terms to include at given order
- do not have to use same regulator
  (eg radial cut-off may be more convenient)
  [Birse and McGovern, nucl-th/0307050]

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Can take cut-off Λ above scale of underlying physics Λ_0

- smaller cut-off artefacts
  (scale of coefficients set by Λ_0 instead of Λ)
- radius of convergence determined by physics: Λ_0
  (can even take $\Lambda \to \infty$?)

Just need to be sure to respect the counting

- renormalise all potentially divergent integrals
- iterate all fixed-point or marginal terms, Q^{-1}
- do not iterate terms that should be perturbations, Q^d with $d \geq 0$
- otherwise . . .

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eg tensor OPE in low partial waves
[Nogga, Timmermans and van Kolck, nucl-th/0506005]

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$\longrightarrow$ if unlucky, lose any consistent counting
eg effective-range term in short-range potential
[Gabbiani, nucl-th/0104088]
or long-range TPE [Pavon Valderrama and Ruiz Arriola, nucl-th/0506047, nucl-th/0507075; Entem *et al*, arXiv:0709.2770]

Deconstructing $^1S_0$ NN scattering 1

Start with distorted-wave effective-range expansion

- iterating OPE justified if we treat λ_{NN} as a low-energy scale
  (or if too lazy to do fourth-order perturbation theory)

$\longrightarrow$ extract effects of OPE from empirical phase shifts $\delta_{\rm PWA}(p)$

- take four good-χ^2 (but old!) Nijmegen analyses:
  PWA93, NijmegenI, NijmegenII, Reid93

Solve radial Schrödinger equation with central OPE

$$-\frac{\mathrm{d}^2u}{\mathrm{d}r^2}+M_N V_{\rm OPE}(r)u(r)=p^2u(r), \qquad p^2=\frac{M_N T_{\rm lab}}{2}$$

$\longrightarrow$ two solutions:
regular $u_R(r)$ ($\rightarrow \sin(pr+\delta_{\rm OPE})$)
and irregular $u_I(r)$ ($\rightarrow -\cos(pr+\delta_{\rm OPE})$)

Use these to construct solution with observed phase shift

$$u(r) = \cos\tilde{\delta}(p)\, u_R(p) - \sin\tilde{\delta}(p)\, u_I(p)$$

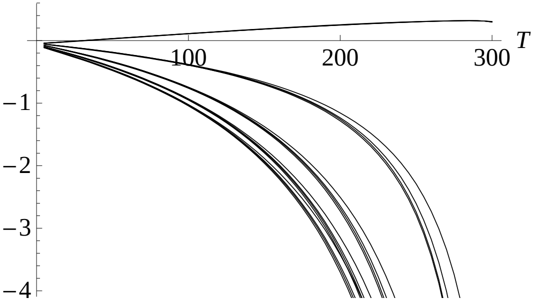
and find short-range potential that generates additional phase $\tilde{\delta}(p) = \delta_{\mathrm{PWA}}(p) - \delta_{\mathrm{OPE}}(p)$

- choose δ-shell form $V_S(r,p) = \frac{1}{4\pi R^2}\,\widetilde{V}_S^{(2)}(p)\,\delta(r-R)$
- take $u(r)$ for $r \geq R$ and $u_R(r)$ for $r \leq R$
- match $u(R) = u_R(R)$ and use discontinuity in derivatives to determine strength

$$\widetilde{V}_S^{(2)}(p) = \frac{4\pi R^2}{M_N}\,\frac{u'(R) - u'_R(R)}{u(R)}$$

(Shukla *et al*: similar philosophy but conical well of radius R that reproduces $u'(R)/u(R)$)

$$\frac{1}{\widetilde{V}_S^{(2)}(p)} + \frac{M_N}{4\pi}\left[\frac{1}{R} - M_N f_{\pi NN}^2 \ln(R\mu)\right]$$ for $R = 1.6,\ 0.8,\ 0.4\ ,0.2,\ 0.1$ fm



Shape converges as $R \to 0$ (to DW effective-range expansion)

Breakdown scale determined by R for large R

($\widetilde{V}_S^{(2)}(p)$ singular at $T_{\rm lab} \simeq 280$ MeV for $R = 1.6$ fm)

Deconstructing $^1S_0$ NN scattering 2

Two-pion exchange

- leading orders $Q^{2,3}$ [Rentmeester et al., nucl-th/9901054]
- plus order Q^2 relativistic correction to OPE [Friar, nucl-th/9901082]
- perturbations: treat to first order → subtract DWBA matrix elements

But matrix elements diverge

→ need to renormalise them first

- cut off radial integrals at R (same as for δ-shell)
- identify and subtract divergent pieces
- use perturbation theory for remaining finite quantities

Strongest divergences from r^{-6} term in order-Q^3 TPE potential and irregular parts of wave functions

- leading terms at each order in energy p^2

$$\int_R^\infty r^2\,\mathrm{d}r\,\frac{1}{r^6}\,u_l(r)^2 \;\sim\; \int_R^\infty r^2\,\mathrm{d}r\,\frac{1}{r^6}\left[\frac{1}{r^2},\,p^2,\,p^4\,r^2,\,p^6\,r^4,\cdots\right]$$
$$\sim\; \frac{1}{R^5},\,\frac{p^2}{R^3},\,\frac{p^4}{R},\,R\,p^6,\cdots$$

- renormalise with counterterms proportional to p^0, p^2, p^4 only
- of orders Q^{-2}, Q^0, Q^2 around nontrivial solution of RG

$\longrightarrow$ terms with orders $d \leq 2$ renormalise order-Q^3 TPE potential

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→ terms with orders $d \leq 2$ renormalise order-Q^3 TPE potential

- power counting works!
  (trivial FP: divergences $\sim R^{-3}$, $R^{-1}p^2$ only → orders Q^0, Q^2)

(Shukla *et al*: did not renormalise, had to keep $R \gtrsim 1$ fm)

Renormalise by subtracting all p^0, p^2, p^4 pieces from integrals

Subtract renormalised matrix element

$$\langle \psi(p) | \, V^{(2)}_{\rm OPE} + V^{(2,3)}_{\rm TPE} + V_{\pi\gamma} | \psi(p) \rangle_{\rm ren}$$

from DW ERE potential $\widetilde{V}^{(2)}_S(p)$
($\longrightarrow$ residual potential containing long-range effects starting at Q^4)

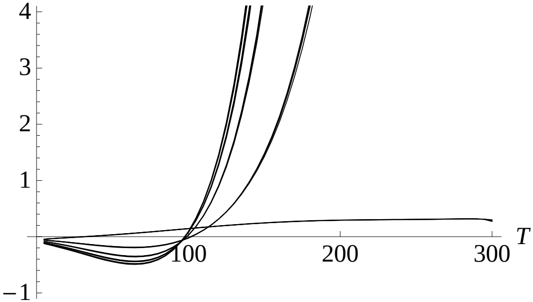
Look at $1/\widetilde{V}_S(p)$ expanded to first order:

$$\frac{1}{\widetilde{V}^{(4)}_S(p)} = \frac{1}{\widetilde{V}^{(2)}_S(p)} + \left(\frac{1}{\widetilde{V}^{(2)}_S(p)}\right)^2 \langle \psi(p) | \, V^{(2)}_{\rm OPE} + V^{(2,3)}_{\rm TPE} + V_{\pi\gamma} | \psi(p) \rangle_{\rm ren}$$

(again, subtract $1/R$ and $\ln R$ terms for convenience in plotting)

Results

For $R = 1.6, 0.8, 0.4, 0.2, 0.1$ fm



- no effect at very low energies since terms up to p^4 subtracted
- p^6 and higher terms grow rapidly above $T = 100$ MeV

$\longrightarrow$ breakdown scale $p \sim 270$ MeV (cf λ_{NN}, $M_\Delta - M_N$)

Discussion

EFT can be used to "deconstruct" empirical phase shifts

- systematically remove effects of known long-range forces

$\longrightarrow$ determine short-range forces directly from data

- extension of Bethe's effective-range expansion
  (here $^1S_0$, previously peripheral singlets and uncoupled triplets)
- terms required by power counting do renormalise divergent matrix elements of TPE potential
- follow power counting $\longrightarrow$ cutoff-independent results as $R \longrightarrow 0$

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But in ${}^1S_0$ channel ...

- expansion seems to break down for $p \gtrsim 270$ MeV
- still need to examine scales in coefficients of p^6, p^8
- coefficient of $r^{-6}\exp(-2m_\pi r)$ contains λ_{NN}, $c_3 \simeq -5$ Gev^{-1}

$\longrightarrow$ “high-energy” scale $\lambda'_{NN} = \left(\dfrac{(16\pi)^2 f_\pi^4}{144 g_A^2 |c_3| M_N}\right)^{1/4} \simeq 115$ MeV

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Trust the RG!

- analyse data following the power counting:
  iterate relevant and marginal terms, Q^d, $d < 0$
  treat irrelevant ones as perturbations, $d \geq 0$
- can then take cutoff above underlying scale
- disentangle physics from artefacts of finite cutoff

$\longrightarrow$ if expansion breaks down: that's physics!
(missing low-energy scales or no separation of scales)

Texas has done one thing; it has invented and established Nuclear EFT, which is an attempt to organise the ignorance of the community, and to elevate it to the dignity of physical force.

[With apologies to Oscar Wilde, The Critic as Artist]