PHYS 30121 NUCLEAR AND PARTICLE PHYSICS

LIGHT HADRONS

The lightest hadrons are those built of of u, d and s quarks (or antiquarks). These can be labelled by their "charges" (baryon and flavour numbers): baryon number B, third component of isospin I_3 and strangeness S. Their electric charge is related to these by

$$Q = I_3 + \tfrac{1}{2}Y,$$

where hypercharge is defined by

$$Y = B + S.$$

Hadrons with the same B and S but different values of I_3 have different charges. If they also have similar internal structures, they have very similar masses, differing by 5 MeV or less. These small splittings result from the small differences between the masses of the u and d quarks and the EM interactions between these quarks. This allows us to group these particles into *isospin multiplets*, with an isospin quantum number I. This can be treated mathematically like an angular momentum (hence the name) with

$$I_3 = +I,\ I-1,\ \ldots,\ -I.$$

Hadrons with similar structures but different strangness have masses that differ by about 150 MeV. This allows us to group them into *supermultiplets* which can be represented by hexagonal or triangular diagrams, where the rows consist of isospin mulitplets.

In the quark model, the lightest mesons are described as $q\overline{q}$ pairs in their ground state, with relative orbital angular momentum $L = 0$ and total quark spin $S_q = 0$. These therefore have total angular momentum $J = 0$. They also have parity $P = -1$ (a consequence of the opposite intrinsic parities of fermions and antifermions) and so they are often called "pseudoscalar mesons". The neutral mesons with zero strangeness are their own antiparticles and so they have definite C parity: $C = (-1)^{L+S_q} = +1$.

The ground-state pseudoscalar mesons are listed below.

particle	S	I	I_3	mass
$\pi^\pm$	0	1	±1	140 MeV
π^0	0	1	0	135 MeV
K^+	$+1$	$\frac{1}{2}$	$+\frac{1}{2}$	494 MeV
K^0	$+1$	$\frac{1}{2}$	$-\frac{1}{2}$	498 MeV
$\overline{K}^0$	-1	$\frac{1}{2}$	$+\frac{1}{2}$	498 MeV
K^-	-1	$\frac{1}{2}$	$-\frac{1}{2}$	494 MeV
η	0	0	0	548 MeV
η'	0	0	0	958 MeV

There are three states with $S = I_3 = 0$, which are superpositions of $u\overline{u}$, $d\overline{d}$ and $s\overline{s}$. One of these, the π^0, is the combination of $u\overline{u}$ and $d\overline{d}$ with total isospin $I = 1$. The other two, the η and η', both have total isospin $I = 0$, with different admixtures of $s\overline{s}$.

This set of mesons is often grouped into a nonet. This can be represented by the following "weight diagram",[1] a plot of hypercharge Y against I_3:

	K^0		K^+	
π^-		π^0, η, η'		π^+
	K^-		$\overline{K}^0$	

However the mass of the η' does not really fit the overall pattern and so this set is often broken up into an octet and a singlet.

The lightest baryons are described as qqq systems in their ground state, with relative orbital angular momentum $L = 0$ and total quark spin $S_q = \frac{1}{2}$. These therefore have total angular momentum $J = \frac{1}{2}$ and, by convention, parity $P = +1$. (Their antiparticles have $J^P = \frac{1}{2}^-$.) The spin and flavour wave functions of baryons with $S_q = \frac{1}{2}$ have a rather complicated structure, with entangled mixtures of symmetric and antisymmetric parts. These baryons are listed below.

particle	S	I	I_3	mass
p	0	$\frac{1}{2}$	$+\frac{1}{2}$	938 MeV
n	0	$\frac{1}{2}$	$-\frac{1}{2}$	940 MeV
Λ	-1	0	0	1116 MeV
Σ^+	-1	1	$+1$	1189 MeV
Σ^0	-1	1	0	1193 MeV
Σ^-	-1	1	-1	1197 MeV
Ξ^0	-2	$\frac{1}{2}$	$+\frac{1}{2}$	1315 MeV
Ξ^-	-2	$\frac{1}{2}$	$-\frac{1}{2}$	1322 MeV

There are two states with $S = -1$ and $I_3 = 0$, corresponding to the quark structure uds. This is because the isospins of the u and d quarks can be added to give either $I = 0$ or $I = 1$. This set of baryons can be grouped into an octet, represented by the following weight diagram:

	n		p	
Σ^-		Σ^0, Λ		Σ^+
	Ξ^-		Ξ^0	

The ground-state hadrons can decay only by weak processes (or EM in the case of the Σ^0), and so they are the most commonly encountered. Their excited states can decay

[1] This term comes from group theory, the area of mathematics that describes symmetries.

strongly and so have much shorter lifetimes, with one important exception: the Ω^- which is the lowest $S = -3$ baryon.

The first excited states of these particles in the quark model have the same orbital structures, but one of the quark spins is flipped to give a larger total spin, and hence a larger J. For the mesons this gives $S_q = 1$ and hence $J^P = 1^-$. The C parity of the neutral states in this supermultiplet is $C = -1$. They are listed below and can be arranged in a nonet, similar to the 0^- mesons.

particle	S	I	mass
$\rho^\pm$, ρ^0	0	1	775 MeV
ω	0	0	783 MeV
K^{*+}, K^{*0}	+1	$\frac{1}{2}$	895 MeV
$\overline{K}^{*0}$, K^{*-}	−1	$\frac{1}{2}$	895 MeV
ϕ	0	0	1019 MeV

The first excited baryons have their quark spins aligned, giving $S_q = \frac{3}{2}$ and hence $J^P = \frac{3}{2}^+$. Their spin wave functions are completely symmetric, as are their flavour wave functions. These baryons are listed below.

particle	S	I	mass
Δ^{++}, $\Delta^\pm$, Δ^0	0	$\frac{3}{2}$	1230 MeV
$\Sigma^{*\pm}$, Σ^{*0}	−1	1	1380 MeV
Ξ^{*0}, Ξ^{*-}	−2	$\frac{1}{2}$	1530 MeV
Ω^-	−3	0	1670 MeV

They can be arranged in a decuplet, represented by the weight diagram:

Δ^-		Δ^0		Δ^+		Δ^{++}
	Σ^{*-}		Σ^{*0}		Σ^{*+}	
		Ξ^{*-}		Ξ^{*0}		
			Ω^-			

The symbols used to denote the particles in this zoo specify their baryon number and strangeness. For example, the kaons (K) are strange mesons, with $B = 0$ and $S = +1$ (or -1 for the antikaons), and the Σs are strange baryons, with $B = 1$ and $S = -1$. From this information and the EM charge of a particle, we can deduce its quark content, as well as other properties such as I_3. The only cases where this does not uniquely specify the quark content are the nonstrange neutral mesons, which can be mixtures of $u\overline{u}$, $d\overline{d}$ and $s\overline{s}$, as mentioned above.

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