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Renormalising nuclear forces

or

How can we build an effective Hamiltonian for nuclear physics?

and other FAQs

Mike Birse
The University of Manchester

INT Program “Effective Field Theories and the Many-Body Problem”, April 2009

What's the point of an effective (field*) theory?

- no model assumptions – just low-energy degrees of freedom and symmetries
- estimates of errors and theory will tell you if it breaks down (no convergence)
- consistency of effective operators and interactions
- effective coupling constants are "universal"

$\longrightarrow$ links between different low-energy phenomena (c_i's: πN scattering $\leftrightarrow$ TPE forces)

$\longrightarrow$ bridges between low-energy observables and underlying theory (scattering lengths: scattering processes $\leftrightarrow$ lattice QCD)

*No creation/destruction of particles $\rightarrow$ just effective quantum mechanics

How does it work?

- systematic expansion in powers of ratios of low-energy scales Q
  (momenta, m_π, ... $\sim$ 200 MeV)
  to scales of underlying physics Λ_0
  (m_ρ, M_N, $4\pi F_\pi$, ... $\gtrsim$ 700 MeV?)

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- works provided we have a consistent expansion
  (otherwise trying to renormalise an infinite number of constants, simultaneously)

Where does it work?

Works well for purely pionic and πN systems

- pions $\sim$ Goldstone bosons of hidden chiral symmetry
  – strong interactions weak at low energies

$\longrightarrow$ chiral perturbation theory

- terms organised by naive dimensional analysis
  aka "Weinberg power counting"
  (simply counts powers of low-energy scales momenta and m_π)

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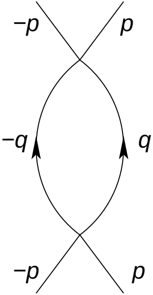
- tendency to circle the wagons and shoot inwards

What's the problem with building an EFT for nuclear forces?

- nucleons interact strongly at low-energies
- simply counting powers of low-energy scales – perturbative
- works for weakly interacting systems (eg pions and photons) but cannot generate bound states (nuclei!)
- need to treat some interactions nonperturbatively

Basic nonrelativistic loop diagram

$$\frac{M}{(2\pi)^3}\int \frac{\mathrm{d}^3 q}{p^2 - q^2 + \mathrm{i}\varepsilon} = -\mathrm{i}\,\frac{M\,p}{4\pi} + \text{analytic}$$



- of order Q [Weinberg (1991)]
  (come back to divergences later)
- better than relativistic case, Q^2
- but potential starts at order Q^0
  (OPE and simplest contact interaction)
- each iteration suppressed by power of Q/Λ_0

$\longrightarrow$ still perturbative (provided $Q < \Lambda_0$)

Workaround: “Weinberg prescription”

- expand potential to some order in Q
- then iterate to all orders in favourite dynamical equation (Schrödinger, Lippmann-Schwinger, . . .)
- widely applied [van Kolck; Epelbaum and Meissner; Machleidt . . .] and even more widely invoked [$\geq$ 9 talks here, so far]

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- widely applied [van Kolck; Epelbaum and Meissner; Machleidt . . .] and even more widely invoked [$\geq$ 9 talks here, so far]
- but no clear power counting for observables
- resums subset of terms to all orders in Q and some of these depend on regulator
- not necessarily a problem if these terms are small
- but what if we rely on them to generate bound states?

How can we iterate interactions consistently?

Identify new low-energy scales

- promote leading-order terms to order Q^{-1}
  (cancels Q from loop $\longrightarrow$ iterations not suppressed)
- can, and must, then be iterated to all orders
  (all other terms: perturbations)

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Examples of new scales

- S-wave scattering lengths $1/a \lesssim 40$ MeV
  [van Kolck; Kaplan, Savage and Wise (1998)]

$\longrightarrow$ for $p \ll m_\pi$: "pionless EFT" $\equiv$ effective-range expansion
[Schwinger (1947); Bethe (1949)]

One-pion exchange

- important for nuclear physics at energies $\sim$ 100 MeV
- order Q^0 in chiral counting

$\longrightarrow$ treat as a perturbation [Kaplan, Savage and Wise (1998)]

- S waves: series coverges slowly, if at all
- OPE "unnaturally" strong
  (cf successes of older phenomenology and Weinberg's scheme)
- strength of OPE set by scale

$$\lambda_{NN} = \frac{16\pi F_\pi^2}{g_A^2 M_N} \simeq 290 \text{ MeV}$$

built out of high-energy scales ($4\pi F_\pi$, M_N) but $\sim 2m_\pi$

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- $\geq$ 4 proposed schemes, $\sim$ 15 years of acrimonious debate

How do we analyse scale-dependence of strongly-interacting systems?

General tool for this: the renormalisation group

- scattering by contact interactions is ill-defined in QM
- couple to virtual states with arbitrarily high momenta
- example: basic loop diagram for S waves behaves as

$$\frac{M}{(2\pi)^3}\int\frac{\mathrm{d}^3q}{p^2-q^2+\mathrm{i}\varepsilon}\sim-\frac{M}{2\pi^2}\int\mathrm{d}q \quad \text{for large } q$$

(linear divergence)

$\longrightarrow$ need to renormalise

Procedure

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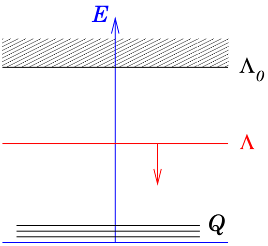

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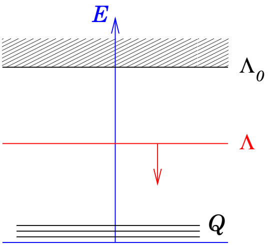

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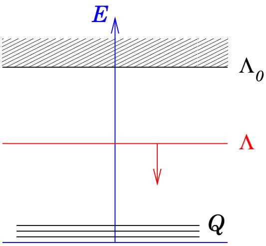

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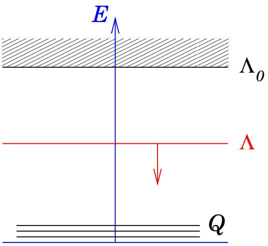

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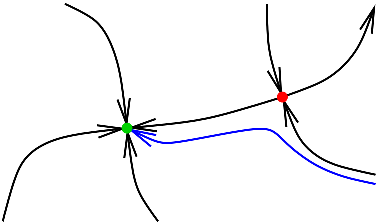
- rescale: express all dimensioned quantities in units of Λ (potential and all low-energy scales)

Follow flow of effective potential as $\Lambda \to 0$

$\longrightarrow$ look for fixed points

- rescaled theories independent of Λ
- correspond to scale-free systems
- endpoints of RG flow



● stable fixed point ● unstable fixed point

Expand around fixed point using perturbations that scale like Λ^{ν}

- $\nu < 0$ relevant or superrenormalisable
  (unstable; eg masses in QFTs)
- $\nu > 0$ irrelevant or nonrenormalisable
  (stable; eg mesonic ChPT)
- $\nu = 0$ marginal or renormalisable
  ($\rightarrow \ln \Lambda$ scale dependence; eg couplings in QED, QCD)

$\longrightarrow$ EFT with power counting: Q^d where $d = \nu - 1$

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Λ is highest acceptable low-energy scale

- order Q
- rescaling $\rightarrow$ power of Λ counts low-energy scales

What does the RG tell us about short-range potentials?

Two fixed points

- trivial $V = 0 \longrightarrow$ free particles
- nontrivial [Birse, McGovern, Richardson (1998)]
  $\longrightarrow$ “unitary limit” (bound state at threshold, $a \longrightarrow \infty$)
- both scale-free systems

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Near trivial fixed point $V(p) = C_0 + C_2\, p^2 + C_4\, p^4 + \cdots$

- energy-dependent: on-shell momentum $p = \sqrt{ME}$
  (come back to momentum dependence)
- p^{2n} are RG eigenfunctions
- orders given by naive (Weinberg) counting: Q^0, Q^2, Q^4, ...
- coefficients C_{2n} related to energy expansion of on-shell K matrix
  (like T matrix but standing-wave bc's – real, analytic)
- appropriate EFT for thermal *np* scattering

Nontrivial fixed point

$$V_0(p,\Lambda) = -\frac{2\pi^2}{M\Lambda}\left[1 - \frac{p}{2\Lambda}\ln\frac{\Lambda+p}{\Lambda-p}\right]^{-1} \quad \text{(sharp cutoff)}$$

- order Q^{-1} (so must be iterated)
- exactly cancels basic loop integral in LS equation

$$\rightarrow \quad T(p) = \mathrm{i}\,\frac{4\pi}{M\,p} \quad \text{(unitary limit)}$$

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Expanding around this point

$$V(p,\Lambda) = V_0(p,\Lambda) + V_0(p,\Lambda)^2\,\frac{M}{4\pi}\left(-\frac{1}{a} + \frac{1}{2}\,r_e\,p^2 + \cdots\right)$$

- factor $V_0^2 \propto \Lambda^{-2}$ promotes terms by two orders compared to naive expectation: Q^{-2}, Q^0, ... [van Kolck; Kaplan, Savage and Wise]
- coefficients of perturbations directly related to observables: effective-range expansion

Enhancement follows from form of wave functions as $r \to 0$

- unitary limit $\longrightarrow$ irregular solutions: $\psi(r) \propto r^{-1}$ (S wave)
- cutoff smears contact interaction over range $R \sim \Lambda^{-1}$

$\longrightarrow$ need extra factor Λ^{-2} to cancel cutoff dependence from $|\psi(R)|^2 \propto \Lambda^2$ in matrix elements of potential

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Other partial waves

- wave functions $\psi(r) \propto r^L$ for small r
  (assuming no low-energy bound state – regular solution)
- extra factor Λ^{2L} needed in potential

$\to$ leading term in L-th partial wave of order Q^{2L}
(Weinberg counting: powers of Q from derivatives of δ-function)

Three-body systems

Attractive: 3 bosons or 3 distinct fermions in unitary limit (triton)

- naive dimensional analysis $\longrightarrow$ leading contact term of order Q^3
- next-to-naive expectation: promoted to Q^1 in unitary limit
  (enhancement of two-body wave functions at small r)
- as hyperradius $R \to 0$ wave functions behave like

$$\psi(R) \propto R^{-2 \pm \mathrm{i} s_0} \qquad s_0 \simeq 1.006 \quad \text{[Efimov (1971)]}$$

$\longrightarrow$ leading three-body force promoted to order Q^{-1}

- marginal perturbation associated with limit cycle of RG
  [Bedaque, Hammer and van Kolck (1999)]

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Repulsive: 1 distinct and 2 identical fermions in unitary limit
(alkali atoms or neutrons)

- hyperradial wave functions $\psi(R) \propto R^{-2+2.1662}$

$\longrightarrow$ leading three-body force of noninteger order $Q^{3.3324}$

How do pion-exchange forces affect the power counting?

Treat λ_{NN} as low-energy scale $\rightarrow$ iterate OPE

Central OPE (spin-singlet waves)

- $1/r$ singularity – not enough to alter power-law forms of wave functions at small r, even if iterated
- $L \geq 1$ waves: weak scattering $\rightarrow$ Weinberg power counting
- $^1S_0$: similar to expansion around unitary fixed point
- except for extra log divergence $\propto m_\pi^2/\lambda_{NN}$
  not distinguishable in practice from leading contact term

$\rightarrow$ KSW-like power counting

Tensor OPE (spin-triplet waves)

- $1/r^3$ singularity
- but higher partial waves protected by centrifugal barrier
- above critical momentum waves resolve singularity
  $\longrightarrow$ OPE not perturbative
- $L \geq 3$: $p_c \gtrsim 2$ GeV $\longrightarrow$ Weinberg counting OK
- $L \leq 2$: $p_c \lesssim 3m_\pi$ $\longrightarrow$ new counting needed
  [Nogga, Timmermans and van Kolck (2005)]

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- wave functions $\psi(r) \propto r^{-1/4}$ multiplied by either sine or exponential function of $1/\sqrt{\lambda_{NN} r}$

$\longrightarrow$ leading contact interaction of order $Q^{-1/2}$ in P, D waves
(very weakly irrelevant)
and of order $Q^{-3/2}$ in ${}^3S_1$–${}^3D_1$ (relevant)

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Contact interaction ("c_E")

- counting still not known:
  need to solve 3-body problem with $1/r^3$ potentials
- expect to be promoted, but by less than in pionless EFT

$\longrightarrow$ order Q^d, $0 < d < 3$?

So, how should we build an effective Hamiltonian?

To order Q^3 (N2LO in Weinberg's counting)

Order	NN	NNN
Q^{-1}	$^1S_0$, $^3S_1$ C_0's, LO OPE	
$Q^{-1/2}$	^3P_J, ^3D_J C_0's	
Q^0	$^1S_0$ C_2	
$Q^{1/2}$	$^3S_1$ C_2	
Q^1		$^1S_0$ C_{D0} OPE
$Q^{3/2}$	^3P_J, ^3D_J C_2's	$^3S_1$ C_{D0} OPE
Q^2	$^1S_0$ C_4, $^1P_1$ C_0, NLO OPE, LO TPE	
$Q^{5/2}$	$^3S_1$ C_4	^3P_J, ^3D_J C_{D0}'s OPE
Q^3	NLO TPE	$^3S_1$ C_{D2} OPE, LO 3N TPE
$Q^?$		C_E

- orange terms absent from "N2LO chiral potential"
- red terms absent from "N3LO"
- order Q^{-1}: have to iterate, order $Q^{-1/2}$: may be better to

What does a finite cutoff do?

- regulates divergences
- also introduces artefacts $\propto \Lambda^{-n}$
  (except for dimensional regularisation)
- suppose only have expansion of effective potential above

$$V(p,\Lambda) = -\frac{2\pi^2}{M\Lambda} - \frac{\pi^3}{M\Lambda^2 a} + \frac{\pi^3}{2M\Lambda^2} r_e p^2 - \frac{2\pi^2}{M\Lambda^3} p^2 + \cdots$$

- last term $\propto p^2$ but of order Q^{-1} (really part of fixed point)
- dominates over effective range term if $\Lambda < \Lambda_0 \sim 1/r_e$

$\longrightarrow$ theory breaks down at momentum scale Λ not Λ_0
size of errors due to truncation determined by $1/\Lambda$ not $1/\Lambda_0$

- keep Λ as large as possible: $\Lambda \gtrsim \Lambda_0$

What about momentum dependence?

Momentum-dependent perturbations (off-shell form of potential)

- trivial FP: same order as corresponding energy-dependent ones

$\longrightarrow$ no cost to trading energy- for momentum-dependence
(field redefinition or "using the equation of motion")

- unitary FP: one order higher

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Possible issues for purely momentum-dependent potentials

- unnaturally strong off-shell behaviour

$\longrightarrow$ will affect other effective operators, 3-body forces, ...

- off-shell T matrix not RG invariant
(cf $V_{\mathrm{low}-k}$ derived from invariance of half-off-shell T matrix)

$\longrightarrow$ no clear power counting for potential or other operators

- probably not problems provided Λ is kept large: $\Lambda \gtrsim \Lambda_0$

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Yes, but only if you keep your hands clean and respect the counting

- renormalise all potentially divergent integrals
- iterate all fixed-point or marginal terms, order Q^{-1}
- do not iterate irrelevant terms, order Q^d with $d \geq 0$
- otherwise . . .

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$\longrightarrow$ more generally, lose any consistent counting
eg effective-range term in short-range potential
[Phillips, Beane and Cohen (1997); and many others]

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Yes, but only if you are very careful . . .

- resumming subset of higher-order terms
- without the counterterms needed to renormalise them
- dangerous: can alter form of short-distance wave functions and destroy power counting (or, at best, change it)
- but problems don't arise, provided higher-order terms are small
- general way to ensure this: keep cutoff small, $\Lambda < \Lambda_0$

Combining EFT and standard many-body methods

Combining EFT and standard many-body methods $\longrightarrow$ tension!

- desire to minimise artefacts, esp for momentum-dependent potentials $\longrightarrow \Lambda \gtrsim \Lambda_0$
- desire to plug full potential into dynamical equation $\longrightarrow \Lambda < \Lambda_0$

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- desire to plug full potential into dynamical equation $\longrightarrow \Lambda < \Lambda_0$

$\longrightarrow$ one way out: take the largest cutoff you dare (just below Λ_0) and stick with it?

- but can't then check for cutoff independence or use cutoff dependence to estimate errors
- already see examples of this in potentials of Epelbaum and Meissner, Entem and Machleidt: $\Lambda \sim 500 - 600$ MeV

Where does all this leave us?

Clear power counting rules for most partial waves, with iterated OPE

- controlled by forms of wave functions as $r \longrightarrow 0$
- in general, not naive dimensional analysis!
- what is counting for 3-body forces in presence of tensor OPE?
- critical momenta for tensor OPE in ${}^3P_J$, ${}^3D_J$ waves with $m_\pi \neq 0$?
- is counting same for waves where tensor OPE is repulsive?

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Contact interactions directly related to “observables” (phase shifts)

- distorted-wave K matrix $\widetilde{K}(p) = -\dfrac{4\pi}{Mp}\tan\big(\delta_{\rm PWA}(p) - \delta_{\rm OPE}(p)\big)$
- either DWBA: expand $\widetilde{K}(p)$ in powers of energy (peripheral w’s)
- or DW effective-range expansion: expand $1/\widetilde{K}(p)$ (S waves)

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In S waves with low-energy bound/virtual states (close to unitary limit)

- energy dependence is lower order than momentum dependence

Uses of EFT potentials in many-body calculations torn between

- keeping cutoff large to minimise artefacts,
  especially if potential is forced to be energy-independent
- and keeping cutoff small so that full potential can be iterated,
  without large higher-order terms destroying the power counting

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- loses much of power of EFT: ability to check cutoff independence,
  or to use cutoff dependence to estimate theoretical errors
- can't have it all!