MANCHESTER 1824

The University of Manchester

Deconstructing nucleon-nucleon scattering

Michael C Birse
The University of Manchester

- Background
  effective field theories for nuclear forces
- Renormalisation group
  tool to determine power counting
- Deconstructing $^1S_0$ scattering
  extracting short-range interactions from empirical phase shifts

Background

Effective field theories

- promise a systematic treatment of strong interactions at low energies
- expansion in powers of ratios of low-energy scales Q (momenta, m_π, ... $\sim$ 200 MeV) to scales of underlying QCD physics Λ_0 (m_ρ, M_N, $4\pi F_\pi$, ... $\gtrsim$ 700 MeV?)
- interactions with ranges $\sim 1/\Lambda_0$ not resolved at scales Q

$\longrightarrow$ replaced by contact interactions

“Weinberg”/naive/engineering power counting

- simply counts powers of low-energy scales Q
- works for weakly interacting systems
  (eg chiral perturbation theory for mesons)
- cannot generate low-energy bound states or resonances
- need to iterate some interactions to all orders
- how do we do this consistently?

First application of ChPT to nuclear forces [Weinberg (1991)]

- nonrelativistic NN loops of order Q (not Q^2)
- potential starts at order Q^0
  (one-pion exchange and simplest contact interaction)

$\longrightarrow$ still perturbative

First application of ChPT to nuclear forces [Weinberg (1991)]

- nonrelativistic NN loops of order Q (not Q^2)
- potential starts at order Q^0
  (one-pion exchange and simplest contact interaction)

$\longrightarrow$ still perturbative

Weinberg-van Kolck scheme

- expand potential to some order in Q
- then iterate to all orders in dynamical equation
  (Schrödinger, Lippmann-Schwinger, . . .)
- widely applied [van Kolck, Epelbaum, Meissner, . . .]
- but no clear power counting for observables

Better: identify new low-energy scales

- promote leading-order terms to order Q^{-1}
  (cancels Q from loop $\longrightarrow$ iterations not suppressed)

$\longrightarrow$ can, and must, then be iterated to all orders
  (all other terms: perturbations)

Better: identify new low-energy scales

- promote leading-order terms to order Q^{-1}
  (cancels Q from loop $\longrightarrow$ iterations not suppressed)

$\longrightarrow$ can, and must, then be iterated to all orders
  (all other terms: perturbations)

Examples of new scales

- S-wave scattering lengths $1/a \lesssim 40$ MeV
  [van Kolck; Kaplan, Savage and Wise (1998)]

$\longrightarrow$ for $p << m_\pi$: "pionless EFT" $\equiv$ effective-range expansion
  [Bethe (1949)]

One-pion exchange

- order Q^0 in chiral counting

$\longrightarrow$ treat as a perturbation [Kaplan, Savage and Wise (1998)]

- S waves: series coverges slowly, if at all
- OPE "unnaturally" strong

  (cf successes of older phenomenology and Weinberg's scheme)
- strength of OPE set by scale

$$\lambda_{NN} = \frac{16\pi F_\pi^2}{g_A^2 M_N} \simeq 290 \text{ MeV}$$

built out of high-energy scales ($4\pi F_\pi$, M_N) but $\sim 2m_\pi$

$\longrightarrow$ another low-energy scale?

Other schemes:

- iterate OPE in chiral limit ($m_\pi = 0$)
  then expand perturbatively in powers of m_π [Beane et al (2001)]
- iterate tensor OPE in attractive spin-triplet waves

$\longrightarrow$ new power counting for contact interactions
[Nogga, Timmermans and van Kolck (2005)]

($\geq$ 4 proposed schemes, $\sim$ 15 years of acrimonious debate)

Other schemes:

- iterate OPE in chiral limit ($m_\pi = 0$)
  then expand perturbatively in powers of m_π [Beane et al (2001)]
- iterate tensor OPE in attractive spin-triplet waves

$\longrightarrow$ new power counting for contact interactions
[Nogga, Timmermans and van Kolck (2005)]

($\geq$ 4 proposed schemes, $\sim$ 15 years of acrimonious debate)

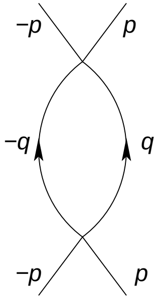
General tool to analyse dependence on low-energy scales
and determine power counting: renormalisation group

Renormalisation group

Scattering by contact interactions ill-defined $\longrightarrow$ need to renormalise

- couple to virtual states with arbitrarily high momenta
- basic loop diagram for S waves behaves as

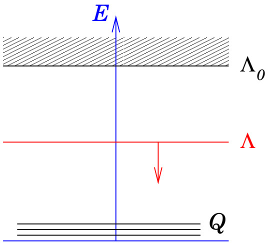
$$M\int\frac{q^2\,\mathrm{d}q}{p^2-q^2}\sim -M\int \mathrm{d}q \quad \text{for large } q$$



Procedure

- identify all relevant low-energy scales Q

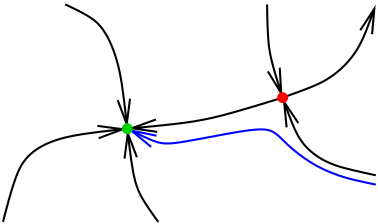
- cut off at arbitary scale Λ between Q and Λ_0 (assumes good separation of scales)
- demand that physics be independent of Λ (eg T-matrix)



- rescale: express all dimensioned quantities in units of Λ (potential and all low-energy scales)

Follow flow as $\Lambda \to 0$; look for fixed points

- rescaled theories independent of Λ
- correspond to scale-free systems
- endpoints of RG flow



● stable fixed point ● unstable fixed point

Expand around fixed point using perturbations that scale like Λ^ν

- $\nu < 0$ relevant or superrenormalisable
  (unstable; eg masses in QFTs)
- $\nu > 0$ irrelevant or nonrenormalisable
  (stable; eg mesonic ChPT)
- $\nu = 0$ marginal or renormalisable
  ($\longrightarrow \ln \Lambda$ scale dependence; eg couplings in QED, QCD)

$\longrightarrow$ EFT with power counting: Q^d where $d = \nu - 1$
(rescaling $\longrightarrow$ power of Λ counts low-energy scales)

RG for short-range potential

General S-wave contact interaction, in momentum space

$$V(k', k, p) = C_{00} + C_{20}(k^2 + k'^2) + C_{02}\, p^2 \cdots$$

- k, k': initial and final relative momenta
- energy-dependence in terms of the on-shell momentum $p = \sqrt{ME}$

RG for short-range potential

General S-wave contact interaction, in momentum space

$$V(k', k, p) = C_{00} + C_{20}(k^2 + k'^2) + C_{02}\, p^2 \cdots$$

- k, k': initial and final relative momenta
- energy-dependence in terms of the on-shell momentum $p = \sqrt{ME}$

Rescaled potential $\hat{V} = M\Lambda V/(2\pi^2)$ satisfies RG equation

$$\Lambda \frac{\partial \hat{V}}{\partial \Lambda} = \hat{k}' \frac{\partial \hat{V}}{\partial \hat{k}'} + \hat{k} \frac{\partial \hat{V}}{\partial \hat{k}} + \hat{p} \frac{\partial \hat{V}}{\partial \hat{p}} + \hat{V} + \hat{V}(\hat{k}', 1, \hat{p}, \Lambda) \frac{1}{1 - \hat{p}^2} \hat{V}(1, \hat{k}, \hat{p}, \Lambda)$$

$\longrightarrow$ two fixed points (independent of Λ)

- trivial $\hat{V} = 0 \longrightarrow$ perturbative power counting (Weinberg)
- nontrivial [Birse, McGovern, Richardson (1998)]

Solution to RG equation near nontrivial fixed point

$$\frac{1}{V(p,\Lambda)} = -\frac{M}{2\pi^2}\left[\Lambda - \frac{p}{2}\ln\frac{\Lambda+p}{\Lambda-p}\right] - \frac{M}{4\pi}\left[-\frac{1}{a} + \frac{1}{2}r_e p^2 + \cdots\right]$$

- first term: fixed point of RG (bound state at zero energy)
- RG eigenvalues $\nu = -1, +1, \ldots$
  correspond to $Q^{-2}, Q^{0}, \ldots$ (shifted by -2 from naive)
  [van Kolck; Kaplan, Savage and Wise (1998)]
- coefficients of perturbations directly related to observables:
  effective-range expansion
- power counting for potential → counting for observables

Similar results in presence of long-range potential of order Q^{-1}
[Barford and Birse (2002), Birse (2007)]

- short-range potential related to distorted-wave observables
- trivial fixed point $\longrightarrow$ DWBA expanded in powers of energy
- nontrivial fixed point $\longrightarrow$ DW effective-range expansion
- power counting determined by singularity of potential as $r \longrightarrow 0$

Similar results in presence of long-range potential of order Q^{-1}
[Barford and Birse (2002), Birse (2007)]

- short-range potential related to distorted-wave observables
- trivial fixed point $\longrightarrow$ DWBA expanded in powers of energy
- nontrivial fixed point $\longrightarrow$ DW effective-range expansion
- power counting determined by singularity of potential as $r \longrightarrow 0$
- Coulomb or Yukawa: similar to counting for pure short-range (except for a log divergence $\longrightarrow$ marginal term in ERE)

$\longrightarrow$ KSW-like counting for short-range potential in ${}^1S_0$ channel

Similar results in presence of long-range potential of order Q^{-1}
[Barford and Birse (2002), Birse (2007)]

- short-range potential related to distorted-wave observables
- trivial fixed point $\longrightarrow$ DWBA expanded in powers of energy
- nontrivial fixed point $\longrightarrow$ DW effective-range expansion
- power counting determined by singularity of potential as $r \longrightarrow 0$
- Coulomb or Yukawa: similar to counting for pure short-range
  (except for a log divergence $\longrightarrow$ marginal term in ERE)

$\longrightarrow$ KSW-like counting for short-range potential in $^1S_0$ channel

- $L > 0$ spin-singlet channels: Weinberg counting
  (no low-lying resonances)

Similar results in presence of long-range potential of order Q^{-1}
[Barford and Birse (2002), Birse (2007)]

- short-range potential related to distorted-wave observables
- trivial fixed point $\longrightarrow$ DWBA expanded in powers of energy
- nontrivial fixed point $\longrightarrow$ DW effective-range expansion
- power counting determined by singularity of potential as $r \to 0$
- Coulomb or Yukawa: similar to counting for pure short-range (except for a log divergence $\longrightarrow$ marginal term in ERE)

$\longrightarrow$ KSW-like counting for short-range potential in $^1S_0$ channel

- $L > 0$ spin-singlet channels: Weinberg counting (no low-lying resonances)
- tensor OPE $\sim 1/r^{-3}$ as $r \to 0$ $\longrightarrow$ NTvK counting in spin-triplet channels $L \leq 2$

Deconstructing $^1S_0$ NN scattering 1

- iterate OPE (justified if we treat λ_{NN} as a low-energy scale)
- use distorted-wave effective-range expansion to extract effects of OPE from empirical phase shifts $\delta_{\rm PWA}(p)$
  (four good-χ^2 Nijmegen analyses: PWA93, NijmegenI, NijmegenII, Reid93)

Deconstructing $^1S_0$ NN scattering 1

- iterate OPE (justified if we treat λ_{NN} as a low-energy scale)
- use distorted-wave effective-range expansion to extract effects of OPE from empirical phase shifts $\delta_{\rm PWA}(p)$
  (four good-χ^2 Nijmegen analyses: PWA93, NijmegenI, NijmegenII, Reid93)

Solve radial Schrödinger equation with central OPE

$$-\frac{\mathrm{d}^2u}{\mathrm{d}r^2} + M_N V_{\rm OPE}(r)u(r) = p^2u(r), \qquad p^2 = \frac{M_N T_{\rm lab}}{2}$$

$\longrightarrow$ regular solution: $u_R(r)$ ($\to \sin(pr + \delta_{\rm OPE})$)
and irregular: $u_I(r)$ ($\to -\cos(pr + \delta_{\rm OPE})$)

Use these to construct solution with observed phase shift

$$u(r) = \cos\tilde{\delta}(p)\, u_R(p) - \sin\tilde{\delta}(p)\, u_I(p)$$

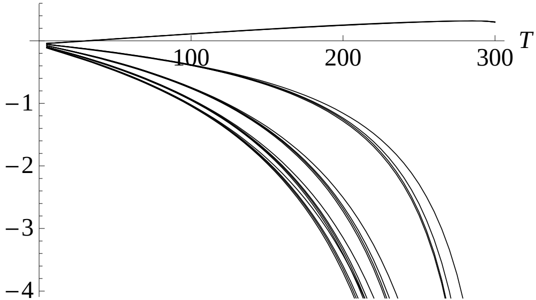
and find short-range potential that generates additional phase $\tilde{\delta}(p) = \delta_{\text{PWA}}(p) - \delta_{\text{OPE}}(p)$

- choose δ-shell form $V_S(r,p) = \frac{1}{4\pi R^2}\, \widetilde{V}_S^{(2)}(p)\, \delta(r-R)$
- take $u(r)$ for $r \geq R$ and $u_R(r)$ for $r \leq R$
- match $u(R) = u_R(R)$ and use discontinuity in derivatives to determine strength

$$\widetilde{V}_S^{(2)}(p) = \frac{4\pi R^2}{M_N}\, \frac{u'(R) - u'_R(R)}{u(R)}$$

[Shukla *et al* (2008): similar philosophy but conical well of radius R]

$$\frac{1}{\widetilde{V}_S^{(2)}(p)} + \frac{M_N}{4\pi}\left[\frac{1}{R} - M_N f_{\pi NN}^2 \ln(R\mu)\right]$$ for $R = 1.6, 0.8, 0.4, 0.2, 0.1$ fm



Linear ($1/R$) and log ($m_\pi^2 \ln(R)$) divergences correspond to those in KSW counting, removed for ease of plotting

Shape converges as $R \to 0$ (to DW effective-range expansion)

Deconstructing $^1S_0$ NN scattering 2

Two-pion exchange

- leading orders $Q^{2,3}$ [Kaiser et al (1997), Rentmeester et al (1999)]
- plus order-Q^2 relativistic correction to OPE [Friar 91999)] and $\pi\gamma$-exchange van Kolck et al (1996)]
- perturbations: treat at first order $\longrightarrow$ subtract DWBA matrix elements

Deconstructing $^1S_0$ NN scattering 2

Two-pion exchange

- leading orders $Q^{2,3}$ [Kaiser et al (1997), Rentmeester et al (1999)]
- plus order-Q^2 relativistic correction to OPE [Friar 91999)] and $\pi\gamma$-exchange van Kolck et al (1996)]
- perturbations: treat at first order $\longrightarrow$ subtract DWBA matrix elements

But matrix elements diverge

$\longrightarrow$ need to renormalise them first

- cut off radial integrals at R (same as for δ-shell)
- identify and subtract divergent pieces
- use perturbation theory for remaining finite quantities

Strongest divergences from r^{-6} term in order-Q^3 TPE potential and irregular parts of wave functions

- leading terms at each order in energy p^2

$$\int_R^\infty r^2\,\mathrm{d}r\,\frac{1}{r^6}\,u_l(r)^2 \sim \int_R^\infty r^2\,\mathrm{d}r\,\frac{1}{r^6}\left[\frac{1}{r^2},\, p^2,\, p^4\,r^2,\, p^6\,r^4,\cdots\right]$$
$$\sim \frac{1}{R^5},\,\frac{p^2}{R^3},\,\frac{p^4}{R},\,R\,p^6,\cdots$$

- renormalise with counterterms proportional to p^0, p^2, p^4 only
- of orders Q^{-2}, Q^0, Q^2 around nontrivial solution of RG

$\longrightarrow$ terms with orders $d \leq 2$ renormalise order-Q^3 TPE potential

Strongest divergences from r^{-6} term in order-Q^3 TPE potential and irregular parts of wave functions

- leading terms at each order in energy p^2

$$\int_R^\infty r^2\,\mathrm{d}r\,\frac{1}{r^6}\,u_l(r)^2 \sim \int_R^\infty r^2\,\mathrm{d}r\,\frac{1}{r^6}\left[\frac{1}{r^2},\,p^2,\,p^4\,r^2,\,p^6\,r^4,\cdots\right]$$
$$\sim \frac{1}{R^5},\,\frac{p^2}{R^3},\,\frac{p^4}{R},\,R\,p^6,\cdots$$

- renormalise with counterterms proportional to p^0, p^2, p^4 only
- of orders Q^{-2}, Q^0, Q^2 around nontrivial solution of RG

$\rightarrow$ terms with orders $d \leq 2$ renormalise order-Q^3 TPE potential

- power counting works

  (trivial FP: divergences $\sim R^{-3}$, $R^{-1}p^2$ only $\rightarrow$ orders Q^0, Q^2)

Renormalise by subtracting all p^0, p^2, p^4 pieces from integrals

Subtract renormalised matrix element

$$\langle\psi(p)|\, V_{\mathrm{OPE}}^{(2)} + V_{\mathrm{TPE}}^{(2,3)} + V_{\pi\gamma}|\psi(p)\rangle_{\mathrm{ren}}$$

from DW ERE potential $\widetilde{V}_S^{(2)}(p)$
($\longrightarrow$ residual potential containing long-range effects starting at Q^4)

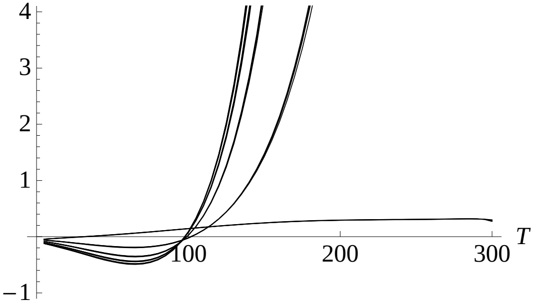
Look at $1/\widetilde{V}_S(p)$ expanded to first order:

$$\frac{1}{\widetilde{V}_S^{(4)}(p)} = \frac{1}{\widetilde{V}_S^{(2)}(p)} + \left(\frac{1}{\widetilde{V}_S^{(2)}(p)}\right)^2 \langle\psi(p)|\, V_{\mathrm{OPE}}^{(2)} + V_{\mathrm{TPE}}^{(2,3)} + V_{\pi\gamma}|\psi(p)\rangle_{\mathrm{ren}}$$

(again, subtract $1/R$ and $\ln R$ terms for convenience in plotting)

Results

For $R = 1.6, 0.8, 0.4, 0.2, 0.1$ fm



- no effect at very low energies since terms up to p^4 subtracted
- p^6 and higher terms grow rapidly above $T = 100$ MeV

$\longrightarrow$ breakdown scale $p \sim 270$ MeV (cf λ_{NN}, $M_\Delta - M_N$)

Summary

Trust the RG!

- identify low-energy scales and find a fixed point
- determine power counting around that point
- then use resulting EFT to “deconstruct” data:
  either DWBA (peripheral waves)
  or DW effective-range expansion (S waves)

Summary

Trust the RG!

- identify low-energy scales and find a fixed point
- determine power counting around that point
- then use resulting EFT to "deconstruct" data:
  either DWBA (peripheral waves)
  or DW effective-range expansion (S waves)
- can use different regulator
  (eg radial cut-off may be more convenient)
- can take cutoff above underlying scale
  (disentangle physics from artefacts of finite cutoff)
  but do not try to iterate perturbations
  [cf Phillips, Beane and Cohen (1997)]

$\longrightarrow$ if expansion breaks down: that's physics!
(missing low-energy scales or no separation of scales)

RG provides power countings to use with iterated OPE

- $^1S_0$: KSW-like
- $L > 0$ spin-singlet channels: Weinberg (perturbative)
- $L \leq 2$ spin-triplet channels: NTvK

$\longrightarrow$ terms required by power counting do renormalise divergent matrix elements of TPE potential

RG provides power countings to use with iterated OPE

- ${}^1S_0$: KSW-like
- $L > 0$ spin-singlet channels: Weinberg (perturbative)
- $L \leq 2$ spin-triplet channels: NTvK

$\longrightarrow$ terms required by power counting do renormalise divergent matrix elements of TPE potential

But in ${}^1S_0$ channel …

- expansion seems to break down for $p \gtrsim 270$ MeV
- still need to examine scales in coefficients of p^6, p^8
- coefficient of $r^{-6}\exp(-2m_\pi r)$ contains λ_{NN}, $c_3 \simeq -5$ GeV^{-1}

$\longrightarrow$ "high-energy" scale

$$\lambda'_{NN} = \left(\frac{(16\pi)^2 f_\pi^4}{144 g_A^2 |c_3| M_N}\right)^{1/4} \simeq 115 \text{ MeV}$$

$\longrightarrow$ need to include Δ?