Deconstructing triplet nucleon-nucleon scattering

Michael C Birse

The University of Manchester

Work in progress, based on:

Birse and McGovern, nucl-th/0307050

and DW renormalisation group:

Barford and Birse, hep-ph/0206146

Birse, nucl-th/0507077

Background

"Weinberg–van Kolck scheme"

[Weinberg (1991); van Kolck et al.; Epelbaum and Meissner; Kaiser; . . .]

- organise terms using perturbative "Weinberg" power counting
- simply count powers of low-energy scales Q

  leading order potential, Q^0: contact term and one-pion exchange

  (nonrelativistic NN loops of order Q, not Q^2)
- need to iterate leading potential in S-waves (e.g. for deuteron)
- requires further IR enhancement

$\longrightarrow$ promote leading-order terms to order Q^{-1} (IR fixed point)

What new low-energy scales justify this?

Also, iterated tensor OPE in spin-triplet channels with $L \neq 0$

– strong cut-off dependence from bound states of attractive tensor potential

$\longrightarrow$ leading short-distance terms need to be promoted from order Q^{2L} to leading order

[Nogga, Timmermans and van Kolck nucl-th/0506005; see also: Epelbaum and Meissner, nucl-th/0609037]

Breakdown of Weinberg counting?

Also, iterated tensor OPE in spin-triplet channels with $L \neq 0$

– strong cut-off dependence from bound states of attractive tensor potential

$\longrightarrow$ leading short-distance terms need to be promoted from order Q^{2L} to leading order

[Nogga, Timmermans and van Kolck nucl-th/0506005; see also: Epelbaum and Meissner, nucl-th/0609037]

Breakdown of Weinberg counting?

Yes!

Also, iterated tensor OPE in spin-triplet channels with $L \neq 0$

– strong cut-off dependence from bound states of attractive tensor potential

$\longrightarrow$ leading short-distance terms need to be promoted from order Q^{2L} to leading order

[Nogga, Timmermans and van Kolck nucl-th/0506005; see also: Epelbaum and Meissner, nucl-th/0609037]

Breakdown of Weinberg counting?

Yes! But there is a viable—consistent and useful—alternative!
(to take up challenge of Epelbaum and Meissner)

What are the low-energy scales for NN scattering?

Momenta, pion mass m_π (obviously for ChPT)

In S-waves: $1/a$ (low-energy bound/virtual states)

What are the low-energy scales for NN scattering?

Momenta, pion mass m_π (obviously for ChPT)

In S-waves: $1/a$ (low-energy bound/virtual states)

Strength of OPE set by scale

$$\lambda_\pi = \frac{16\pi F_\pi^2}{g_A^2 M_N} \simeq 290 \text{ MeV}$$

Built out of high-energy scales ($4\pi F_\pi$, M_N) but $\sim 2m_\pi$

What are the low-energy scales for NN scattering?

Momenta, pion mass m_π (obviously for ChPT)

In S-waves: $1/a$ (low-energy bound/virtual states)

Strength of OPE set by scale

$$\lambda_\pi = \frac{16\pi F_\pi^2}{g_A^2 M_N} \simeq 290 \text{ MeV}$$

Built out of high-energy scales ($4\pi F_\pi$, M_N) but $\sim 2m_\pi$

Identify λ_π as additional low-energy scale (cf. Weinberg: $1/M_N \sim Q$)

$\longrightarrow$ leading OPE of order Q^{-1} (fixed point)

$\longrightarrow$ iterate OPE as in WvK scheme

Power counting

Schrödinger equation for spin-triplet channels at short distances

- dominated by $1/r^3$ singularity of tensor OPE
- solutions satisfy (uncoupled waves)

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2}+\frac{2}{r}\frac{\mathrm{d}}{\mathrm{d}r}-\frac{L(L+1)}{r^2}-\frac{\beta_{LJ}}{r^3}\right]\psi_0(r)=0 \qquad \left(\beta\propto\frac{1}{\lambda_\pi}\right)$$

$\longrightarrow$ Bessel functions for $\beta<0$ (modified Bessel if $\beta>0$)

$$\psi_0(r)\propto r^{-1/2}\left[\sin\alpha\, J_{2L+1}\left(2\sqrt{\frac{|\beta_{LJ}|}{r}}\right)+\cos\alpha\, Y_{2L+1}\left(2\sqrt{\frac{|\beta_{LJ}|}{r}}\right)\right]$$

α: fixes phase of short-distance oscillations (self-adjoint extension or leading short-distance parameter)

Perturbative treatment of tensor potential

- only possible if waves cannot resolve $1/r^3$ singularity
- must be well below critical momentum: $p_c \sim [L(L+1)]^{3/2}/|\beta|$
  (to avoid region where OPE $\gtrsim$ centrifugal)
- $p_c \lesssim 3m_\pi$ for $L \leq 2$ but $p_c \gtrsim 2$ GeV for $L \geq 3$

$\longrightarrow$ OPE nonperturbative in S-, P-, D-waves for momenta $\sim m_\pi$
perturbation theory should be valid for F-waves and above

Perturbative treatment of tensor potential

- only possible if waves cannot resolve $1/r^3$ singularity
- must be well below critical momentum: $p_c \sim [L(L+1)]^{3/2}/|\beta|$
  (to avoid region where OPE $\gtrsim$ centrifugal)
- $p_c \lesssim 3m_\pi$ for $L \leq 2$ but $p_c \gtrsim 2$ GeV for $L \geq 3$

$\longrightarrow$ OPE nonperturbative in S-, P-, D-waves for momenta $\sim m_\pi$
perturbation theory should be valid for F-waves and above

To renormalise S-, P-, D-waves up to lab energies $T \sim 300$ MeV

- need to use cut-off in nonperturbative regime

$\longrightarrow$ renormalisation-group flow controlled by Bessel functions of $\sqrt{|\beta|/r}$ (tensor OPE), not usual $(pr)^L$ forms (centrifugal)

RG analysis with running cutoff applied to DW's of $1/r^3$ potential

$\longrightarrow$ new power counting [nucl-th/0507077]

– leading contact interaction of order $Q^{-1/2}$ (weakly irrelevant)

– higher energy-dependent terms at orders $Q^{3/2}$, $Q^{7/2}$, ...

– same counting for both attractive and repulsive tensor potentials

– corresponds to DWBA amplitude expanded in powers of energy

RG analysis with running cutoff applied to DW's of $1/r^3$ potential

$\longrightarrow$ new power counting [nucl-th/0507077]

- leading contact interaction of order $Q^{-1/2}$ (weakly irrelevant)
- higher energy-dependent terms at orders $Q^{3/2}$, $Q^{7/2}$, ...
- same counting for both attractive and repulsive tensor potentials
- corresponds to DWBA amplitude expanded in powers of energy

For values of α that give low-energy bound state/resonance ($\alpha \simeq \pi/2$)

- leading contact interaction of order $Q^{-3/2}$ (relevant)
- corresponds to DW effective-range expansion
- relevant to $^3S_1$–$^3D_1$ and possibly $^3P_0$
  (other waves: only for very narrow ranges of α)

Deconstruction 1

Use distorted-wave methods (DWBA or DW ERE)

– extract effects of known long-range potentials from empirical phase shifts

– examine sizes, energy dependences of residual amplitudes [nucl-th/0307050; see also Epelbaum and Meissner]

$\longrightarrow$ define DW K-matrix (p: on-shell CM momentum)

$$\widetilde{K}(p) = -\frac{4\pi}{Mp}\tan\Big(\delta_{\rm PWA}(p) - \delta_{\rm OPE}(p)\Big)$$

“Data”: $\delta_{\rm PWA}(p)$ from four good-χ^2 (but old!) Nijmegen analyses

– PWA93, NijmegenI, NijmegenII, Reid93 (uncoupled *np* waves: $^3P_0$, $^3P_1$, $^3D_2$, $^3F_3$, $^3G_4$)

Short-range potential: energy-dependent δ-shell form

$$V_S(p,r) = \frac{1}{4\pi R_0^2 |\psi_0(R_0)|^2}\,\widetilde{V}(p)\,\delta(r-R_0)$$

- to avoid $r \longrightarrow 0$ (DW's either singular or vanishing)
  [contrast Ruiz Arriola and Pavón Valderrama, nucl-th/0504067]
- divide by energy-independent short-distance solution $|\psi_0(R_0)|^2$
  to remove dependence of scattering on arbitrary R_0
- solutions normalised so that for large r they match on to
  short-distance form of free solutions $j_L(pr)/p^L$

$$\psi_0(r) \sim \frac{r^L}{(2L+1)!!} \qquad \text{as } r \longrightarrow \infty$$

recover Weinberg counting in channels where scattering is weak

Weak short-range potentials can be determined directly from $\widetilde{K}(p)$ using DWBA

$$\widetilde{V}(p) = \frac{|\psi_0(R_0)|^2}{|\psi(p,R_0)|^2}\widetilde{K}(p)$$

Leading OPE used for DW $\psi(p,r)$

$\longrightarrow$ residual interaction $\widetilde{V}^{(2)}(p)$ starts at order Q^2
(Weinberg counting!)

Weak short-range potentials can be determined directly from $\widetilde{K}(p)$ using DWBA

$$\widetilde{V}(p) = \frac{|\psi_0(R_0)|^2}{|\psi(p,R_0)|^2}\widetilde{K}(p)$$

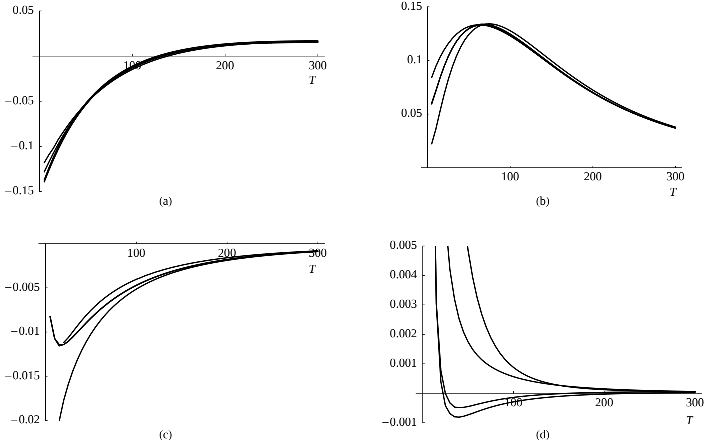
Leading OPE used for DW $\psi(p,r)$

$\longrightarrow$ residual interaction $\widetilde{V}^{(2)}(p)$ starts at order Q^2

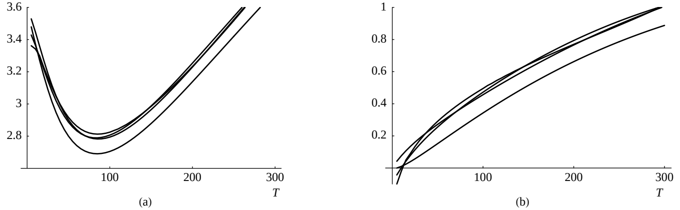
(Weinberg counting!)

Results

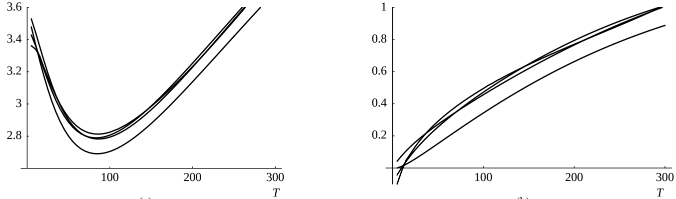
- for lab energies $T \leq 300$ MeV ($p \leq 375$ MeV)
- $R_0 = 0.1$ fm (needs to be $\lesssim 0.6$ fm)
- $\alpha = 0$ in waves where OPE is attractive, except $^3P_0$



Short-distance interaction $\widetilde{V}^{(2)}(p)$ (fm^{2L+2}) against lab kinetic energy T (MeV) for the channels (a) $^3P_1$, (b) $^3D_2$, (c) $^3F_3$, (d) $^3G_4$



$^3P_0$ short-distance interaction $\widetilde{V}^{(2)}(p)$ (fm^4) for two choices of short-distance phase: (a) $\alpha = 0$, (b) $\alpha = 0.54$



(a) (b)

$^3P_0$ short-distance interaction $\widetilde{V}^{(2)}(p)$ (fm^4) for two choices of short-distance phase: (a) $\alpha = 0$, (b) $\alpha = 0.54$

Notes

– strong energy dependences for $T \lesssim 100$ MeV

$\longrightarrow$ need to remove order-$Q^{2,3}$ OPE and TPE

– $^3P_0$ residual interaction very strong

$\longrightarrow$ may need to iterate by taking $\alpha \neq 0$

– uncertainties in PWA's for low energies and large L's

Deconstruction 2

Two-pion exchange starts at order-Q^2

- perturbative $\longrightarrow$ subtract DWBA matrix elements
- use order-$Q^{2,3}$ potentials [Rentmeester et al., nucl-th/9901054]

Corresponding relativistic correction to OPE [Friar, nucl-th/9901082]

- order-Q^2 in Weinberg counting (order-Q^1 in new counting?)

$$V_{1\pi}^{(2)}(r) = -\frac{p^2}{2M^2}\left[V_{1\pi}^{(LO)}(r)\right]$$

Residual short-distance interaction starts at order Q^4

$$\widetilde{V}^{(4)}(p) = \frac{|\psi_0(R_0)|^2}{|\psi(p,R_0)|^2}\left(\widetilde{K}(p) - \langle\psi(p)|V_{1\pi}^{(2)} + V_{2\pi}^{(2,3)} + V_{\pi\gamma}|\psi(p)\rangle\right)$$

DWBA matrix elements diverge at small r

– potentials for $r \longrightarrow 0$ behave as

$$V_{2\pi}^{(3)} \propto \frac{1}{r^6} \qquad V_{1\pi}^{(2)}(r) \propto \frac{p^2}{r^3}$$

– short-distance wave functions $\psi_0(r) \propto r^{-1/4} \sin\left(2\sqrt{|\beta|/r}\right)$

– cut off integrals at $r = R_0$ (same as in V_S)

$\longrightarrow$ energy-independent divergences

(including ones with powers of m_π and λ_π)

one energy-dependent divergence $\propto p^2$

Renormalised by order-$Q^{-1/2}$ and $Q^{3/2}$ contact terms

Up to order Q^4 new counting also contains order-$Q^{7/2}$ term ($\propto p^4$)

Remove all these to leave order-Q^4 interaction in new counting

- fit quadratic in energy to $\widetilde{V}^{(4)}(p)$ over range $T = 100 - 200$ MeV
- subtract

  (still take $R_0 = 0.1$ fm)

Renormalised by order-$Q^{-1/2}$ and $Q^{3/2}$ contact terms

Up to order Q^4 new counting also contains order-$Q^{7/2}$ term ($\propto p^4$)

Remove all these to leave order-Q^4 interaction in new counting

- fit quadratic in energy to $\widetilde{V}^{(4)}(p)$ over range $T = 100 - 200$ MeV
- subtract

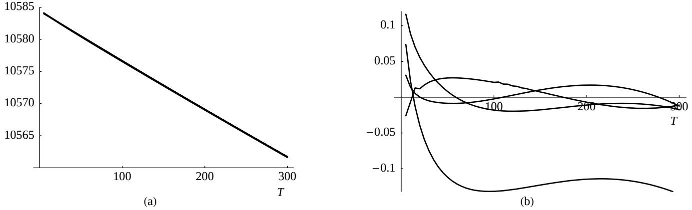
  (still take $R_0 = 0.1$ fm)

Repulsive channel $^3P_1$: no divergences

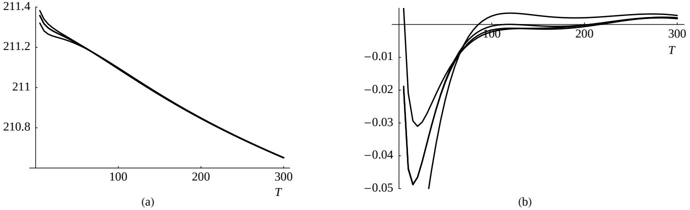
- wave functions $\psi_0(r) \propto r^{-1/4} \exp\left(-2\sqrt{|\beta|/r}\right)$ as $r \to 0$
- matrix elements all finite

  but still depend strongly on R_0 in range $0.1 - 0.6$ fm

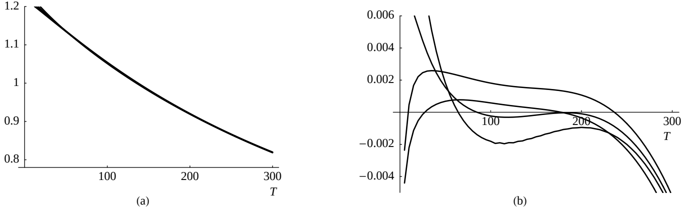
$\longrightarrow$ analyse using new power counting



Short-distance interactions $\widetilde{V}^{(4)}(p)$ (fm^4) for $^3P_0$ with $\alpha = 0.54$
(a) unsubtracted, (b) quadratic fit subtracted



Short-distance interactions $\widetilde{V}^{(4)}(p)$ (fm^6) for $^3D_2$ with $\alpha = 0$



Short-distance interaction $\widetilde{V}^{(4)}(p)$ (fm^4) for $^3P_1$
(a) unsubtracted, (b) quadratic fit subtracted

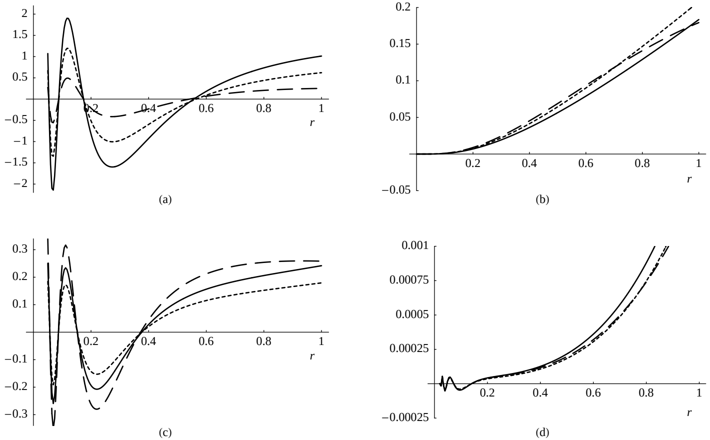
Conclusions (provisional)

Using DWBA method to extract OPE and TPE effects from Nijmegen PWA's, find:

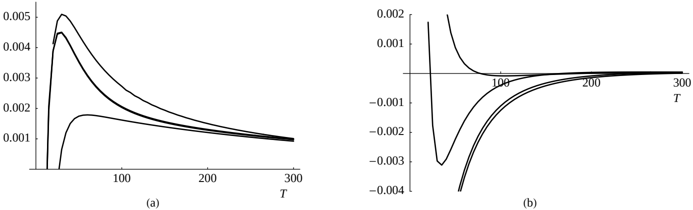
- peripheral waves ${}^3F_3$ and ${}^3G_4$ can be analysed using strict Weinberg counting for $T \lesssim 300$ MeV
  (perturbative OPE and contact terms starting at order Q^{2L})
- ${}^3P_0$ and ${}^3D_2$ need nonperturbative treatment of OPE and new power counting for $T \sim 100$ MeV and cut-offs $R \lesssim 0.6$ fm
  ($\Lambda \gtrsim 300$ MeV): contact terms starting at order $Q^{-1/2}$
- ${}^3P_1$ intermediate but new counting seems to work better
- ${}^3P_0$ leading contact term may need to be iterated
- strong energy dependences for $T \lesssim 100$ MeV well described by order-$Q^{2,3}$ OPE and TPE
- exception is ${}^3D_2$: missing attraction?

Critical relative momenta in chiral limit

Channel	p_c
$^3S_1$–$^3D_1$	66 MeV
$^3P_0$	182 MeV
$^3P_1$	365 MeV
$^3P_2$–$^3F_2$	470 MeV
$^3D_2$	403 MeV
$^3D_3$–$^3G_3$	382 MeV
$^3F_3$	2860 MeV
$^3F_4$–$^3H_4$	2330 MeV
$^3G_4$	1870 MeV



Wave functions $\psi(r)/p^L$ for (a) $^3P_0$, (b) $^3P_1$, (c) $^3D_2$, (d) $^3G_4$.
Short-dashed lines: $T = 5$ MeV; long-dashed lines: $T = 300$ MeV;
solid lines: energy-independent asymptotic form



Short-distance interaction $\widetilde{V}^{(4)}(p)$ (fm^{2L+2}) for (a) $^3F_3$, (b) $^3G_4$

No sign of divergences for $R_0 \gtrsim 0.05$ MeV
(results independent of R_0 for $R_0 \lesssim 0.6$ fm)

$\longrightarrow$ consistent with Weinberg counting
(or zero given uncertainties in $^3G_4$ PWA's)