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Coupled-channel effective field theory and proton-^{7}Li scattering

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Outline

- Background: effective field theories for few-body systems
- Determining power counting from wave functions near origin
  (renormalisation group)
- Extension to coupled channels
- Application to $p + {}^7\text{Li} \leftrightarrow n + {}^7\text{Be}$
  state of ${}^8\text{Be}$ $\sim$ at $n + {}^7\text{Be}$ threshold

Effective field theory

- no model assumptions – just low-energy degrees of freedom and symmetries
- systematic expansion in powers of ratios of low-energy scales Q to scales of underlying physics Λ_0
- interactions with ranges $\sim 1/\Lambda_0$ not resolved at scales Q $\longrightarrow$ replaced by contact interactions
- works provided we have a a good separation of scales
- estimates of errors and theory will tell you if it breaks down (no convergence)
- consistency of effective operators and interactions [cf talk by Higa]

Works well for purely pionic and πN systems

- pions $\sim$ Goldstone bosons of hidden chiral symmetry
  – strong interactions weak at low energies

$\longrightarrow$ chiral perturbation theory

- expansion in ratios of low-energy scales:
  momenta, m_π, typically $\sim$ 200 MeV
  to QCD scales: m_ρ, M_N, $4\pi F_\pi$, ... $\gtrsim$ 700 MeV?
- terms organised by naive dimensional analysis
  aka "Weinberg power counting"
  (simply counts powers of low-energy scales Q)

Extensions to few-body systems

Nuclear EFT

- two- and three-nucleon forces
- long-range parts from ChPT
- short-range: contact interactions

"Cluster EFT" (aka "halo EFT")

- weakly bound nuclei with two- or three-cluster structures
- clusters treated as structureless objects
- assumes excitation energies of clusters $\gg$ energies of interest

Extensions to ≥ 2 nucleons or clusters raise new problems

- nucleons (and nuclei) interact strongly at low energies
- simply counting powers of low-energy scales: perturbative
- works for weakly interacting systems
  (eg pions, photons and ≤ 1 nucleon)

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- basic nonrelativistic loop diagram of order Q [Weinberg (1991)]
- and potential starts at order Q^0 (simplest contact interaction)
- each iteration suppressed by power of Q/Λ_0

$\rightarrow$ still perturbative (provided $Q < \Lambda_0$)

Workaround: “Weinberg prescription”

- expand potential to some order in Q
- then iterate to all orders in favourite dynamical equation (Schrödinger, Lippmann-Schwinger, . . .)
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- but no clear power counting for observables
- resums subset of terms to all orders in Q
  (and some of these depend on regulator)
- not necessarily a problem if these terms are small
- but what if we rely on them to generate bound states?

How to iterate interactions consistently

Identify new low-energy scales

- promote leading-order terms to order Q^{-1}
  (cancels Q from loop $\longrightarrow$ iterations not suppressed)

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Example of new scales in NN system

- S-wave scattering lengths $\longrightarrow$ $1/a \lesssim 40$ MeV

$\longrightarrow$ for $p \ll m_\pi$: "pionless EFT" ($\equiv$ effective-range expansion)
[van Kolck; Kaplan, Savage and Wise (1998)]

Effective-range expansion

Expansion of inverse of on-shell K matrix in powers of energy
(like T matrix but standing-wave bc's – real, analytic)

$$\frac{1}{K(p)} = \frac{M}{4\pi}\left(-\frac{1}{a} + \frac{1}{2}r_e p^2 + \cdots\right)$$

- expansion around "unitary limit" $1/K(p) = 0$
  (bound state at threshold)
- low-energy scale: $1/a \sim Q$
- remaining terms assumed to be "natural": $r_e \sim 1/\Lambda_0$

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$$K(p) = \frac{4\pi}{M}\left(-a - \frac{1}{2}\, r_e\, a^2 p^2 + \cdots\right)$$

$\longrightarrow$ leading term of order Q^{-1}
other terms promoted by two orders in Q

Enhancement follows from form of wave functions as $r \to 0$

- unitary limit $\to$ irregular solutions: $\psi(r) \propto r^{-1}$ (S wave)
- cutoff at scale Λ smears contact interaction over range $R \sim \Lambda^{-1}$

$\longrightarrow$ need extra factor Λ^{-2} in potential to cancel cutoff dependence from $|\psi(R)|^2 \propto \Lambda^2$ in matrix elements
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Complete approach is to use the renormalisation group

- demand observables to be independent of cutoff Λ

$\longrightarrow$ two fixed points (scale-free systems)

- trivial: noninteracting system $\to$ "Weinberg" power counting
- nontrivial: unitary limit $\to$ effective-range expansion
- results agree with simple arguments based on wave functions
(even for systems with $1/r^2$ or $1/r^3$ potentials)

Two coupled channels

Three fixed points of potential (2×2 matrix)

- trivial in both channels (boring)
- bound states at threshold in both channels (very unlikely)
  [Cohen, Gelman and van Kolck (2004)]
- one bound state at threshold

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Last one is of most interest

- starting point for EFT describing systems with a single low-energy bound or virtual state
- need to use effective-range expansion in one channel but Weinberg power counting in the other
- and channels are mixtures of the two asymptotic ones

$\longrightarrow$ can't simply expand either $\mathbf{K}$ or $\mathbf{K}^{-1}$

System with two channels, separated by energy Δ

- S waves, one low-energy state, momenta $p \ll m_\pi$

Small scales Q

- on-shell momenta $p_1 = \sqrt{2M_1 E}, \quad p_2 = \sqrt{2M_2(E-\Delta)}$
  (M_i: reduced masses)
- inverse scattering length in strongly interacting channel $1/a_\alpha$

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"Underlying" high-energy scales Λ_0

- range of forces m_π
- inverse sizes of clusters $1/R_i$
- excitations of clusters $\sqrt{2M_i E_{\mathrm{ex},i}}$

Expand in powers of Q/Λ_0

Asymptotic channels both couple to low-energy state

- strongly interacting channel defined by $\mathbf{u}_\alpha = \begin{pmatrix} \cos\phi \\ \sin\phi \end{pmatrix}$
- weakly interacting channel by orthogonal vector $\mathbf{u}_\beta$

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Parameters of resulting EFT

Order	Parameter
Q^{-1}	large scattering length a_α
	mixing angle ϕ
Q^0	small scattering length a_β
	effective range in strongly interacting channel r_α
Q^1	"off-diagonal" effective range r_m (energy-dependent mixing)

(subleading terms enhanced by two orders in α channel,
off-diagonal by one order, no enhancement in β)

To order Q (NNLO)

$$\mathbf{T} = -2\pi\mathbf{M}^{-1/2}\Bigg\{\Bigg[-\frac{1}{a_\alpha}+\frac{r_\alpha}{2}p_2^2-\mathrm{i}p_\alpha$$

$$-a_\beta\left[\left(1-\mathrm{i}a_\beta p_\beta\right)p_\mathrm{m}^2-\mathrm{i}r_\alpha p_2^2 p_\mathrm{m}\right]\Bigg]^{-1}\mathbf{u}_\alpha\mathbf{u}_\alpha^\dagger$$

$$-a_\beta\left[1-\mathrm{i}a_\beta p_\beta+a_\beta p_\mathrm{m}^2\left(-\frac{1}{a_\alpha}-\mathrm{i}p_\alpha\right)^{-1}\right]\mathbf{u}_\beta\mathbf{u}_\beta^\dagger$$

$$+a_\beta\Bigg[\mathrm{i}p_\mathrm{m}\left[\left(-\frac{1}{a_\alpha}-\mathrm{i}p_\alpha\right)\left(1+\mathrm{i}a_\beta p_\beta\right)+\frac{r_\alpha}{2}p'^2-a_\beta p_\mathrm{m}^2\right]^{-1}$$

$$+\frac{r_\mathrm{m}}{2}p_2^2\left(-\frac{1}{a_\alpha}-\mathrm{i}p_\alpha\right)^{-1}\Bigg]\left(\mathbf{u}_\alpha\mathbf{u}_\beta^\dagger+\mathbf{u}_\beta\mathbf{u}_\alpha^\dagger\right)\Bigg\}\mathbf{M}^{-1/2}$$

Orders of terms follow from RG analysis: r_m multiplies $a_\alpha p_2^2 \sim \mathcal{O}(Q)$

Combinations of momenta above thresholds from finite parts of loop integrals:

$$\begin{aligned}
p_\alpha &= \theta(p_1^2)p_1\cos^2\phi + \theta(p_2^2)p_2\sin^2\phi \\
p_\beta &= \theta(p_1^2)p_1\sin^2\phi + \theta(p_2^2)p_2\cos^2\phi \\
p_\mathrm{m} &= \big(\theta(p_1^2)p_1 - \theta(p_2^2)p_2\big)\sin\phi\cos\phi
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(divergent parts $M\Lambda/(2\pi^2)$ cancelled by renormalisation)

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Charged particles $\longrightarrow$ need to use Coulomb distorted waves

- new low-energy scales: inverse Bohr radii $\kappa_{1,2}$
- no change to basic power counting
  (1/r not singular enough to change behaviour of wave functions)
- replace $\mathrm{i}p_i$ by finite parts of integrals with DWs
  $j_i(p_i) = \kappa_i\left(2h(\eta_i) + \mathrm{i}\,C^2_{\eta_i}/\eta_i\right), \quad \eta_i = \kappa_i/p_i$

$p + {}^7\text{Li} \leftrightarrow n + {}^7\text{Be}$ coupled channels

Ideal test case for EFT

- $n + {}^7\text{Be}$ threshold at $\Delta = 1.6442$ MeV above $p + {}^7\text{Li}$
- $J^P = 2^-$ excited state of ${}^8\text{Be}$ within ~ 10 keV of threshold

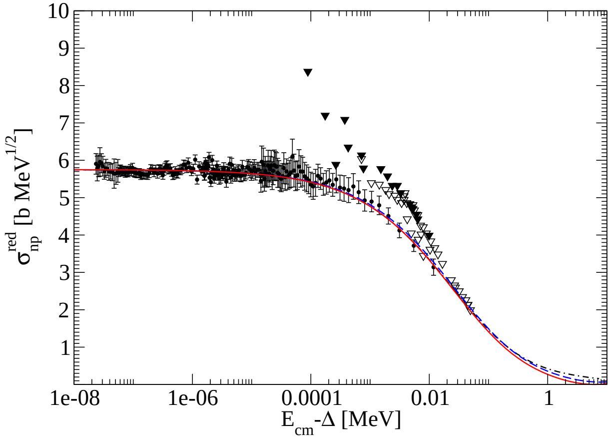
$\rightarrow$ huge cross section for ${}^7\text{Be}(n,p){}^7\text{Li}$ at thermal energies
(crucial for BBN of ${}^7\text{Li}$ but very well measured)

- $\frac{5}{2}^-$ excited states of $A = 7$ nuclei at ~ 7 MeV

Data (and phenomenological analyses)

- $p + {}^7\text{Li}$ ${}^5S_2$ phase shift from PWA and *R*-matrix fits [Brown *et al* (1973)]
- cross section for ${}^7\text{Be}(n,p){}^7\text{Li}$ [Koehler *et al* (1988)]
- *R*-matrix fits [Adahchour and Descouvemont (2003)]

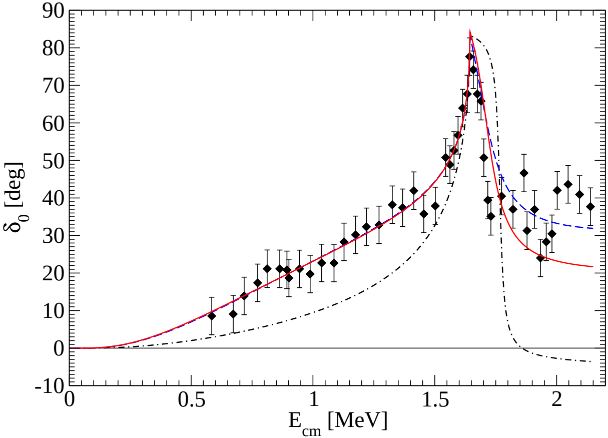
^{7}Be$(n,p)^7$Li reduced cross section $\sigma_{np}^{\text{red}} = \sigma_{np}\sqrt{E-\Delta}$



Circles: (n,p) data from Koehler *et al*; triangles: old data on (p,n)

EFT fits at different orders $\sim$ indistinguishable

$p + {}^7\mathrm{Li}\ {}^5S_2$ phase shift



Diamonds: PWA of Brown *et al* (our estimate of uncertainties)

EFT fits: LO dot-dashed, NLO dashed, NNLO solid

Scattering parameters

Order	a_α [fm]	ϕ	a_β [fm]	r_α [fm]	r_m [fm]
LO	-17.76	46.63°	–	–	–
NLO	-19.37	51.82°	-1.96	3.79	–
NNLO	-19.11	50.45°	-1.07	2.58	-5.42

EFT looks convergent, underlying scale $\Lambda_0 \sim 100$ MeV $(2\text{ fm})^{-1}$

- $\sigma_{np}^{\rm red}$: only threshold value matters
- phase shift below threshold not sensitive to r_m
- PWA above threshold relies on old (p, n) $\longrightarrow$ not fitted

$\longrightarrow$ really only NLO parameters reliable (NNLO χ^2 very flat)

- expansion of T matrix $\longrightarrow$ unitarity not guaranteed

$\longrightarrow$ NNLO fits unstable unless we impose unitarity constraint or use R-matrix results to fix $\sigma_{np}^{\rm red}$ above 0.1 MeV

$J^P = 2^-$ state of ^{8}Be

T matrix pole at complex energy $E = E_r - \mathrm{i}\Gamma/2$ with

$$
\begin{aligned}
E_r &= 1.71\ \text{MeV} = \Delta + 0.07\ \text{MeV} \\
\Gamma &= 0.12\ \text{MeV}
\end{aligned}
$$

- lies on 4th Riemann sheet: $\mathrm{Im}[p_1] > 0$, $\mathrm{Im}[p_2] < 0$
- partial widths $\Gamma_p = 2.47$ MeV, $\Gamma_n = 0.34$ MeV
  ($\sim$ virtual state not a resonance $\rightarrow$ don't add up to Γ)
- similar to results of R-matrix fits

Conclusions

Cluster (halo) EFT applied to coupled-channel systems

- interesting fixed point of RG: single low-energy state
- expansion – mixture of effective-range and perturbation theory
- illustrates enhancements expected from RG
  (short-distance behaviour of wave functions)

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$p + {}^7\mathrm{Li} \leftrightarrow n + {}^7\mathrm{Be}$ ideal candidate

- $J^P = 2^-$ state of ${}^8\mathrm{Be}$ close to $n + {}^7\mathrm{Be}$ threshold
- ${}^7\mathrm{Be}(n,p){}^7\mathrm{Li}$ well described at any order in EFT
- S-wave phase shift below threshold well described at NLO
  (NNLO fit needs better PWA above threshold)

$\longrightarrow$ convergent expansion with underlying scale $\Lambda_0 \sim 100$ MeV

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EFT can provide a more systematic alternative to R-matrix analyses