You may assume the following formulae in any exam question if proof is not explicitly requested (updated Nov 2010):

Spherical Harmonics:

$$Y_0^0(\theta,\phi)=\sqrt{\tfrac{1}{4\pi}} \qquad Y_1^{\pm 1}(\theta,\phi)=\mp\sqrt{\tfrac{3}{8\pi}}\,\sin\theta\,\mathrm{e}^{\pm i\phi} \qquad Y_1^0(\theta,\phi)=\sqrt{\tfrac{3}{4\pi}}\,\cos\theta$$

Harmonic Oscillator:

$$\hat{a}=\frac{1}{\sqrt{2}}\left(\frac{\hat{x}}{x_0}+i\frac{x_0}{\hbar}\hat{p}\right) \quad \text{and} \quad \hat{a}^\dagger=\frac{1}{\sqrt{2}}\left(\frac{\hat{x}}{x_0}-i\frac{x_0}{\hbar}\hat{p}\right) \quad \text{where } x_0=\sqrt{\frac{\hbar}{m\omega}}$$

$$\hat{a}|n\rangle=\sqrt{n}|n-1\rangle \quad \text{and} \quad \hat{a}^\dagger|n\rangle=\sqrt{n+1}|n+1\rangle.$$

Wave functions:

$$\phi_n(x)=(2^n n!)^{-1/2}H_n(\tfrac{x}{x_0})\times(\pi x_0^2)^{-1/4}\exp\bigl(-x^2/(2x_0^2)\bigr)$$

$H_0(z)=1;\ H_1(z)=2z;\ H_2(z)=4z^2-2;\ H_3(z)=8z^3-12z;\ H_4(z)=16z^4-48x^2+12$

Hydrogen atom:

normalised radial wave functions ($\int_0^\infty r^2R^2\mathrm{d}r=1$), where $a_0=\hbar c/(mc^2\alpha)$:

$$R_{10}(r)=2a_0{}^{-3/2}\mathrm{e}^{-r/a_0},\ R_{20}(r)=(2a_0^3)^{-1/2}\left(1-\tfrac{r}{2\,a_0}\right)\mathrm{e}^{-r/2a_0},\ R_{21}(r)=\bigl(24a_0^3\bigr)^{-1/2}\tfrac{r}{a_0}\mathrm{e}^{-r/2a_0}.$$

Radial integrals for the state $|nlm\rangle$:

$$\langle r^2\rangle = \tfrac{1}{2}a_0^2n^2\bigl(5n^2+1-3l(l+1)\bigr), \qquad \langle r\rangle=\tfrac{1}{2}a_0\bigl(3n^2-l(l+1)\bigr), \qquad \langle r^{-1}\rangle=(n^2a_0)^{-1},$$

$$\langle r^{-2}\rangle = \bigl((l+\tfrac{1}{2})n^3a_0^2\bigr)^{-1} \qquad \langle r^{-3}\rangle=\bigl(l(l+\tfrac{1}{2})(l+1)n^3a_0^3\bigr)^{-1}.$$

Landé g-factor: $g_{jls}=1+\bigl(j(j+1)-l(l+1)+s(s+1)\bigr)/\bigl(2j(j+1)\bigr)$

Integrals: for positive integer n,

$$\int_0^\infty x^n\mathrm{e}^{-x/a}\mathrm{d}x=n!\,a^{n+1} \qquad \int_0^\infty x^n\mathrm{e}^{-x^2/a^2}\mathrm{d}x=\begin{cases}(n-1)!!a^{n+1}/2^{(n+1)/2} & \text{if } n \text{ is odd,}\\ (n-1)!!\sqrt{\pi}a^{n+1}/2^{(n/2+1)} & \text{if } n \text{ is even.}\end{cases}$$

Time-independent perturbation theory:

$$|n^{(1)}\rangle=\sum_{m\neq n}\frac{\langle m^{(0)}|\widehat{H}^{(1)}|n^{(0)}\rangle}{E_n^{(0)}-E_m^{(0)}}|m^{(0)}\rangle \qquad \text{and} \qquad E_n^{(2)}=\sum_{m\neq n}\frac{\left|\langle m^{(0)}|\widehat{H}^{(1)}|n^{(0)}\rangle\right|^2}{E_n^{(0)}-E_m^{(0)}}.$$

Time-dependent perturbation theory: If at t_0 a system is in a state $|i\rangle$, to first order the chance of finding it in the state $|n\rangle$ at time t is $|d_n(t)|^2$, where

$$d_n(t)=-\tfrac{i}{\hbar}\int_{t_0}^{t}\mathrm{e}^{i(E_n-E_i)t/\hbar}\langle n|\widehat{H}^{(1)}(t)|i\rangle\mathrm{d}t$$

Fermi's Golden Rule: $R_{i\to n}=\frac{2\pi}{\hbar}\left|\langle n|\widehat{H}^{(1)}|i\rangle\right|^2\delta(E_{ni}-\hbar\omega)$

Scattering theory: Born approximation expression for the scattering amplitude:

$$f(k,\theta,\phi)=\frac{m}{2\pi\hbar^2}\int\mathrm{e}^{i\mathbf{q}\cdot\mathbf{r}}V(\mathbf{r})\mathrm{d}^3\mathbf{r}$$

where $\mathbf{q}=\mathbf{k}_i-\mathbf{k}_f$ is the difference between the initial and final wave vectors.

Phase shifts and cross section: $\sigma=\sum_{l=0}^{\infty}\sigma_l=\sum_{l=0}^{\infty}\frac{4\pi}{k^2}(2l+1)\sin^2\delta_l$