PHYS20352 Statistical Mechanics 2019-20: Notes on examination

This is the first year I have given this course, and only the second year that the course has run in its current form. This document provides some notes on past and (possible) future exams to help with revision. See the end for specifics to 2020.

As always, the tutorial sheets for the course provide the best guide to the examinable material, with the proviso that (i) examples sheets don't contain bookwork and (ii) some examples are clearly too long or tricky to be appropriate for an exam (the ones marked as optional).

The structure is the standard format with a multi-part compulsory question 1 and a choice of 2 from 3 other questions. There will be a formula sheet at the start of the exam with useful integrals and a couple of other things. It is attached at the end of this note. It is designed to be useful in *any* exam, so it is not a guide to what is actually on this year's exam.

In 2017/18 and for decades before, the material of this course was spread over two courses, PHYS20352 Thermal and Statistical Physics and PHYS30151 Bose and Fermi Gases. About half the material of PHYS20352 now resides in PHYS10352, and will only be examined in the current course in the context of statistical physics (no heat engines, for instance). So everything on old PHYS20352 exams should be familiar to you. There was quite a bit of repetition between the two courses, so in fact almost all of PHYS30151 is also covered in the current course and almost all questions are accessible. The exceptions are those on the properties of graphene, though as graphene can be treated as a 2D ultrarelativistic Fermi gas with zero chemical potential, many question parts even on graphene remain relevant.

The density of states may be denoted $\dfrac{\mathrm{d}n}{\mathrm{d}k}$, $D(k)$ or $g(k)$ (and similarly for $g(\varepsilon)$). It always includes the volume or area, so it is a density per unit k but not per unit volume. In that the usage differs from PHYS30252 Solid State.

The following are lists of questions from past exams I wouldn't set (which isn't to say that ideas within them, such as Maxwell relations, cannot be invoked):

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- 2019: 1b
- 2018: 1c; 2
- 2017 1b; 2; 3b,c
- 2016 1a,b,c; 2
- 2015 1b,c, 3b

PHYS30151

Note that in 2016 and before, the lecturer set $k_B \to 1$, that is T has units of energy and entropy and heat capacities are dimensionless. Restoring the missing factors of k_B is a good exercise! Graphene featured more heavily then than it would now.

- 2018 all OK
- 2017 1e
- 2016 2c (I'd phrase the rest of the question in a more appropriate way given we haven't studied graphene)
- 2015 1c; 2c

2020

For 2020, the formative exam will consist of 3 compulsory questions, all of the usual length. The first will be a standard five-part question 1. The second two will be each have two roughly equal parts, so that together they cover the bulk of the syllabus. There will be no bookwork in questions 2&3 (there isn't much point in an open-book exam).

Previous examiners of PHYS20352 have been fond of the classical calculation of the partition function (section 3.13) but I will not examine that this year. Similarly one of the previous examiners of PHYS30151 is fond of equilibrium among elementary particles in plasmas or the early universe (section 3.12.1) but again I will not examine this topic this year. I have not seem anything else in past exams which, though on the syllabus, is off-limits due to strikes or COVID-19 this year.

Formula sheet

Information which may be used in this paper:

The density of states for a spinless particle in two dimensions is $g(k) = \dfrac{Ak}{2\pi}$.

The symbol β is defined as $\beta = 1/(k_B T)$.

The symbol n_Q is defined as $n_Q = \left(\dfrac{mk_B T}{2\pi\hbar^2}\right)^{3/2}$

Stirling's approximation is $\ln N! \approx N \ln N - N$ for $N \gg 1$.

$$\sum_{n=0}^{\infty} (n+1)\, x^n = (1-x)^{-2} = \text{for } |x| < 1.$$

The following integrals may be useful (note $n!! \equiv n(n-2)(n-4)\ldots 1$ for odd n.)

$$\int_0^\infty \frac{x^{1/2}}{e^x+1}dx = 0.678094 \qquad \int_0^\infty \frac{x^{1/2}}{e^x-1}dx = 2.31516$$

$$\int_0^\infty \frac{x}{e^x+1}dx = \frac{\pi^2}{12} \qquad \int_0^\infty \frac{x}{e^x-1}dx = \frac{\pi^2}{6}$$

$$\int_0^\infty \frac{x^{3/2}}{e^x+1}dx = 1.15280 \qquad \int_0^\infty \frac{x^{3/2}}{e^x-1}dx = 1.78329$$

$$\int_0^\infty \frac{x^2}{e^x+1}dx = 1.80309 \qquad \int_0^\infty \frac{x^2}{e^x-1}dx = 2.40411$$

$$\int_0^\infty \frac{x^{5/2}}{e^x+1}dx = 3.08259 \qquad \int_0^\infty \frac{x^{5/2}}{e^x-1}dx = 3.74453$$

$$\int_0^\infty \frac{x^3}{e^x+1}dx = \frac{7\pi^4}{120} \qquad \int_0^\infty \frac{x^3}{e^x-1}dx = \frac{\pi^4}{15}$$

$$\int_0^\infty x^n e^{-x/a}dx = n!\, a^{n+1} \qquad \int_0^\infty x^n e^{-x^2/a^2}dx = \frac{(n-1)!!\sqrt{\pi}a^{n+1}}{2^{(n/2+1)}} \quad \text{for even } n.$$